Off-Path Beliefs

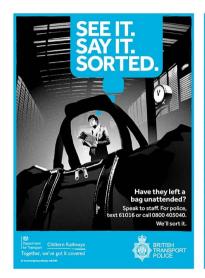
Duarte Gonçalves
University College London

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VU Amsterdam

UCL

November 2025







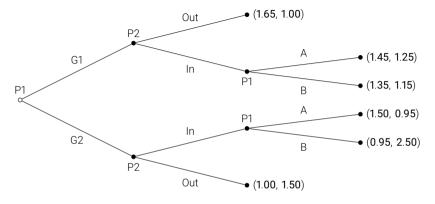
Gonçalves (UCL)

... you typically infer that something won't be right

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Why do we care about beliefs about off-path?

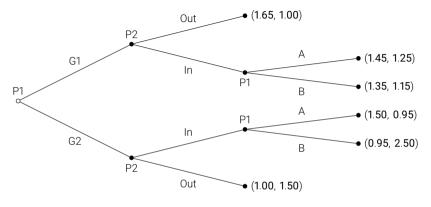
How Many Subgames?



P1 prefers A to B in both G1 and G2. P2 then prefers In in G1 and Out in G2. P1 then prefers G1.

Unique equilibrium (G1, A, A) and (In, Out); indeed, unique NE outcome G1-In-A

How Many Subgames?



P1 prefers A to B in both G1 and G2. P2 then prefers In in G1 and Out in G2. P1 then prefers G1.

Unique equilibrium (G1, A, A) and (In, Out); indeed, unique NE outcome G1-In-A

But... if P2 sees G2 (off-path move), what to think about follow on moves? In or Out?

Depends on beliefs about what P1 does after... Stick to believing P1 chooses A despite mistake?

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- Standard strategy for instructors is to gloss over this and focus on the appeal of subgame perfection But it's an issue for soln concepts in dynamic games in general, incl. NE and rationalisability

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Why do we care about beliefs about off-path?

- Solution concepts and their predictions crucially hinge on beliefs about what doesn't happen off-path
- Standard strategy for instructors is to gloss over this and focus on the appeal of subgame perfection But it's an issue for soln concepts in dynamic games in general, incl. NE and rationalisability
- If I observe off-path behaviour, should I still use the same model of how the other makes choices? Beliefs about others' off-path behaviour affect my choices (and ultimately determine what is actually off-path)

Nov 2025 4 Off-Path Reliefs

What would we like to know?

- What do people think is actually off-path?
 - Does it agree with (some) our notion(s) of off-path?

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 - Do they ascribe meaning to/make inferences from them?
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What do we know?

Surprisingly little (none?) research on this fundamental question

Literature

Theory: Is sequential rationality reasonable when observing others' making mistakes (Kreps & Wilson, 1982; Reny, 1992)?

Off-Path behaviour, such as in centipede games (McKelvey & Palfrey, 1992; Palacios-Huerta & Volij, 2009; Levitt, List, & Sadoff, 2011) or in games with (non-)credible threats (Goeree & Holt, 2001)

Belief elicitation...

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about others' one-shot choices: Huck & Weizsäcker (2002)
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about others' beliefs: Agranov & Detkova (2025)

in sequential/repeated games: Nyarko & Schotter (2002); Hyndman et al. (2012); Danz, Fehr, & Kübler (2012); Wang (2018)

about own future mistakes: Chakraborty & Kendall (2025)



Experimental Design

Experimental framework to study off-path beliefs

Elicit beliefs about choices at terminal info set

Compare to beliefs about choices same info set when on-path vs off-path

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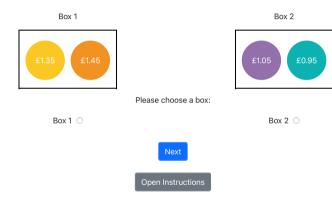
Compare to beliefs about choices same info set when on-path vs off-path

1. Elicit choices first...

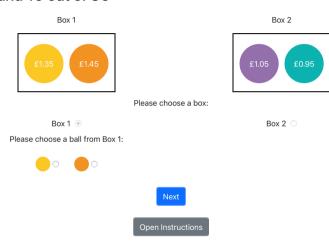
Round 1 out of 36



Round 10 out of 36



Round 10 out of 36



Careful construction of boxes

Box = (Min, Max) = (Min, Min + Spread)

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Min = £0.45, £0.95, £1.35

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All combinations: 9 boxes; All pairwise combinations: 45 pairs

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Two parts

Part 1: 9 rounds, one for each box, random order

Part 2: 27 rounds, random subset of pairs of boxes

1 round randomly selected for payment

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Choices

WYSIWYG payment + £2.55 completion

225 participants. Avg pay £4.35; Median duration 7min

Experimental framework to study off-path beliefs

Elicit beliefs about choices at terminal info set

Compare to beliefs about choices same info set when on-path vs off-path

- 1. Elicited choices first (experiment 1)
- 2. Elicit beliefs (experiment 2)

Experimental framework to study off-path beliefs

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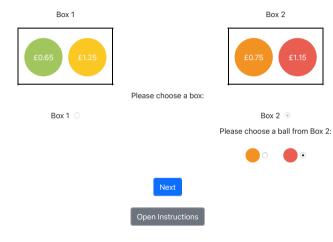
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Explain other's task (practice)

Elicit beliefs

Practice Round of the Other Participants' Task



Practice Round of the Other Participants' Task



In the practice round, you chose Box 2 and the

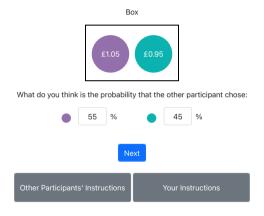
If this were a paid round and the other participant had made this choice, they would be paid a bonus of £1.15.

ball.

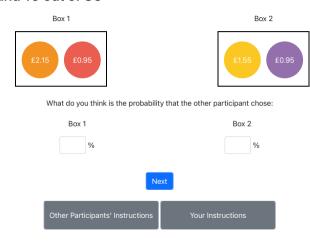
Click 'Next' to see your instructions for this study.

Next

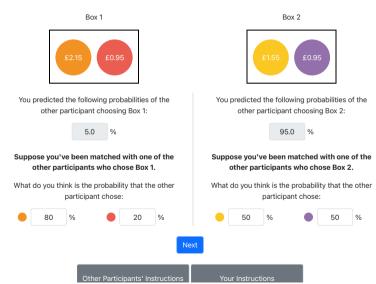
Round 6 out of 36



Round 10 out of 36



Round 10 out of 36



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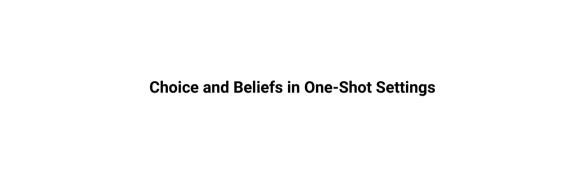
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1 round randomly selected for payment

Beliefs

BSR £3 vs 0 + £3 completion

225 participants. Avg pay £5.70; Median duration 17min



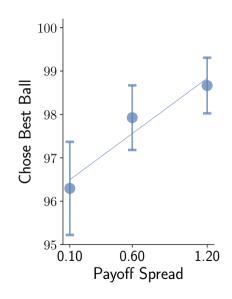
More Indifferent ⇒ More Mistakes

+ Spread

⇒ Less indifferent

→ Fewer mistakes

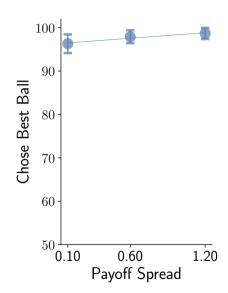
(Figs: 95% CI with clustered se's at participant level)



More Indifferent ⇒ More Mistakes

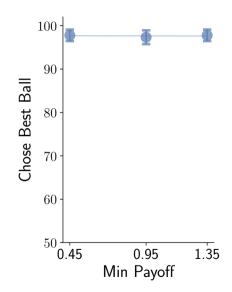
But do note the *y*-axis...

Basically very few mistakes anyway (<2.5%)



Level Shifts Don't Matter

No effect of min i.e., adding a constant has no effect

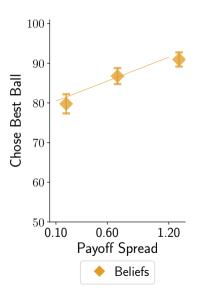


More Indifferent ⇒ Believe More Mistakes

+ Spread

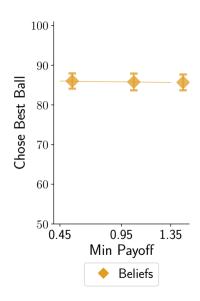
⇒ Less indifferent

⇒ Believe in fewer mistakes



Believe that Level Shifts Don't Matter

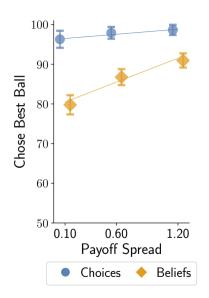
Also no effect of min i.e., adding a constant has no effect



Beliefs vs Choices

Overestimate others' mistakes

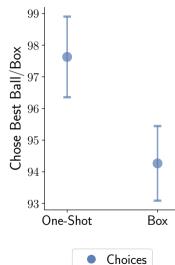
Beliefs react more to fundamentals than choices





Is there an Off-Path?

Barely: on avg, chose off-path <6%



What Makes Off-Path Off-Path?

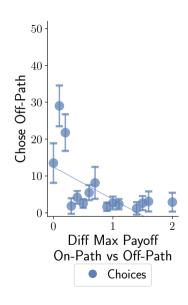
Larger difference in max payoff

⇒ Off-Path worse

⇒ Less indifferent

⇒ Fewer mistakes/off-path choices

(On avg off-path <6%)



What Makes Off-Path Off-Path?

Smaller difference in min payoff

⇒ Off-Path relatively safer

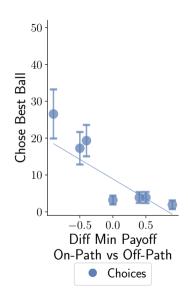
⇒ Fewer mistakes/off-path choices

Difference in min negative

⇒ Off-Path min > On-Path min

(by definition, Off-Path max < On-Path max)

'Trade-off' gain vs risk? (temptation/self-control problem?)





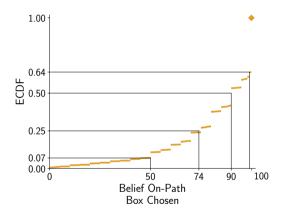
Yes. (i.e., much more than I thought they/I would)

Do People Actually Believe There's an Off-Path?

Yes. (i.e., much more than I thought they/I would)

36% of instances: on-path wp 1

>50% of instances: on-path wp \geq 90%



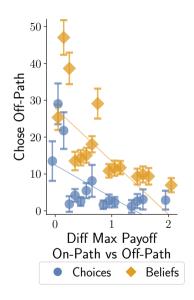
What Determines Beliefs in the Off-Path?

Larger difference in max payoff

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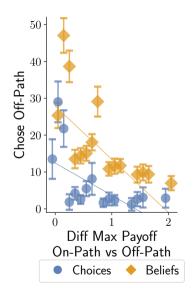
⇒ Off-Path worse

⇒ Less indifferent

⇒ Believe fewer mistakes/off-path choices

Note: almost looks like level shift

Beliefs react more to fundamentals than choices



What Determines Beliefs in Off-Path?

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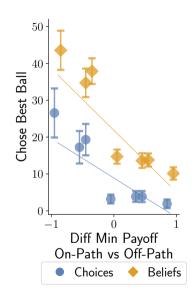
⇒ Believe fewer mistakes/off-path choices

Difference in min negative

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Believe in others' sophistication regarding their own self-control?





What Happens Off-Path?

iid Trembles? Irrelevant mistakes?

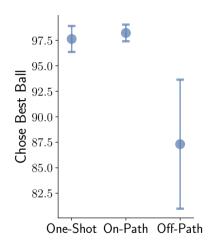
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Greater likelihood of mistake following Off-Path

On-Path mistakes even less frequent

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Correlated Mistakes

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Significant individual heterogeneity of propensity to mistakes throughout

 $(\chi^2$ test independence: mistakes in one-shot, on-path, choosing off-path)

Correlated Mistakes

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On-Path mistakes even less frequent

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Significant individual heterogeneity of propensity to mistakes throughout

 $(\chi^2 \mbox{ test independence: mistakes in one-shot, on-path, choosing off-path)}$

Also: Mistakes in One Shot predictive of choosing Off-Path:

Freq (Choosing Off-Path | No Mistake in One-Shot) = 96%

Freq (Choosing Off-Path | Made Mistake in One-Shot) = 76%

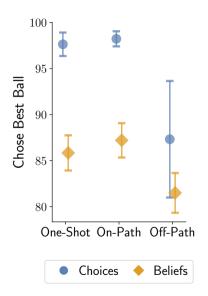
Corr (Nr. Choices Off-Path, Nr. Made Mistake in One-Shot) = .6

Beliefs On- and Off-Path

Believe in greater likelihood of mistake following Off-Path
On-Path infer mistakes slightly less frequent
Off-Path infer mistakes somewhat more frequent

Recall: not necessarily the case irrelevant mistakes, more mistakes from subsequent choices

Interestingly: now beliefs seem to react less than choices



Underinfer from off-path choices: correlation neglect

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Grether regression (80)

log-odds Belief Mistake | Off-Path = β_1 log-odds Belief Mistake + β_2 log-likelihood Off-Path

Bayesian inference: $\beta_1 = \beta_2 = 1$

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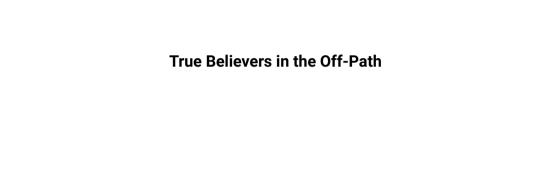
Indeed, for choices:

log-odds Choice Mistake | Off-Path = log-odds Choice Mistake in One-Shot + log-likelihood Off-Path

Estimates: base-rate neglect $\beta_1 \approx .9$ and severe underinference $\beta_2 \approx .2$

Note: under-reaction of beliefs specifically about off-path correlation, not general feature:

e.g., beliefs do react much more than choices to differences in payoffs



Who Truly Believes There's an Off-Path?

Surprisingly many people!

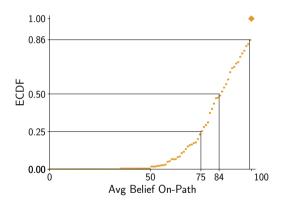
Who *Truly* Believes There's an Off-Path?

Surprisingly many people!

14% of true believers: *always* assign prob 1 to on-path

Median participant:

on avg believes on-path chosen 84% of the time



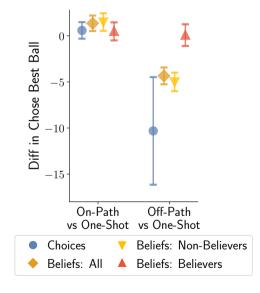
Updating from Zero Probability Event

"Equilibrium believers": believe others play on-path and behave coherently, both on and off path

Always assign prob. 1 to on-path and almost always optimal behaviour, both on and off-path

Conditional on on- or off-path event, true believers don't revise their model

(from diff-in-diff, also via Grether regression)





(In this very simple setting) Off-path is actually almost always off-path...

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... but if you see something that doesn't look right, you should typically infer something

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Most people

- (1) overpredict noise,
- (2) underinfer from off-path behaviour, neglecting correlation in mistakes

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... but if you see something that doesn't look right, you should typically infer something

Most people

- (1) overpredict noise,
- (2) underinfer from off-path behaviour, neglecting correlation in mistakes

True believers in on-path behaviour are true believers:

believe mistakes never happen, and, when they do, keep believing they just won't happen again

Heterogeneity: apart from faster people inferring slightly more, other dimensions not significant (age, uni, etc)

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THANK YOU!

Overview

- 1. Experimental Design
 - Choices
 - Beliefs
- 2. Choice and Beliefs in One-Shot Settings
 - Choice Mistakes
 - Beliefs about Mistakes
- 3. The Off-Path
 - Choosing the Off-Path
 - Believing in the Off-Path
- 4. On the Off-Path
- 5. True Believers in the Off-Path
- 6. Take-aways