# Off-Path Beliefs

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- Standard strategy for instructors is to gloss over this and focus on the appeal of subgame perfection

  But it's an issue for soln concepts in dynamic games in general, incl. NE and rationalisability

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  But it's an issue for soln concepts in dynamic games in general, incl. NE and rationalisability
- If I observe off-path behaviour, should I still use the same model of how the other makes choices?

  Beliefs about others' off-path behaviour affect my choices

  (and ultimately determine what is actually off-path)

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**What do we know?**

Surprisingly little (none?) research on this fundamental question

# Literature

**Theory:** Is sequential rationality reasonable when observing others' making mistakes (Kreps & Wilson, 1982; Reny, 1992)?

**Off-Path *behaviour***, such as in centipede games (McKelvey & Palfrey, 1992; Palacios-Huerta & Volij, 2009; Levitt, List, & Sadoff, 2011) or in games with (non-)credible threats (Goeree & Holt, 2001)

**Belief elicitation...**

about others' *one-shot choices*: Huck & Weizsäcker (2002)

about others' *beliefs*: Agranov & Detkova (2025)

in sequential/repeated games: Nyarko & Schotter (2002); Hyndman et al. (2012); Danz, Fehr, & Kübler (2012); Wang (2018)

about *own future mistakes*: Chakraborty & Kendall (2025)

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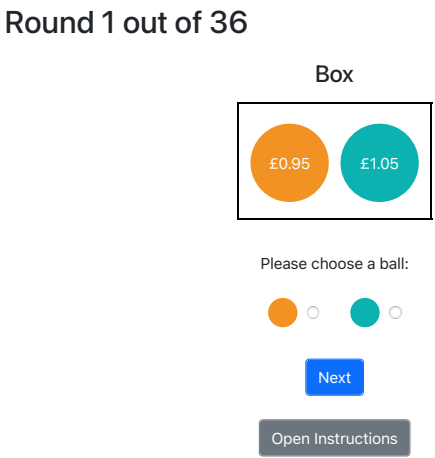
# Experimental Design: Choices



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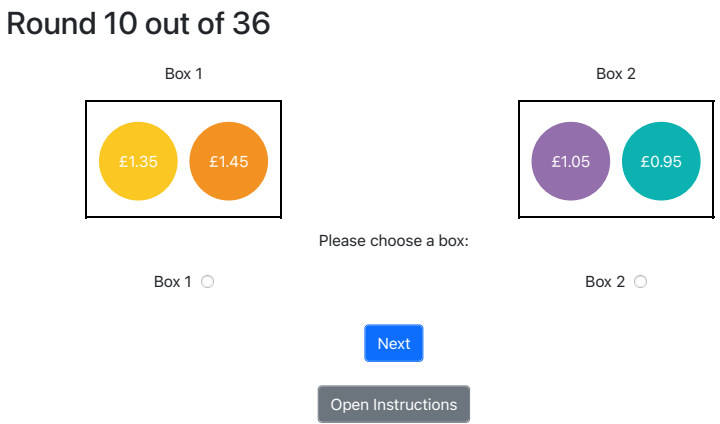

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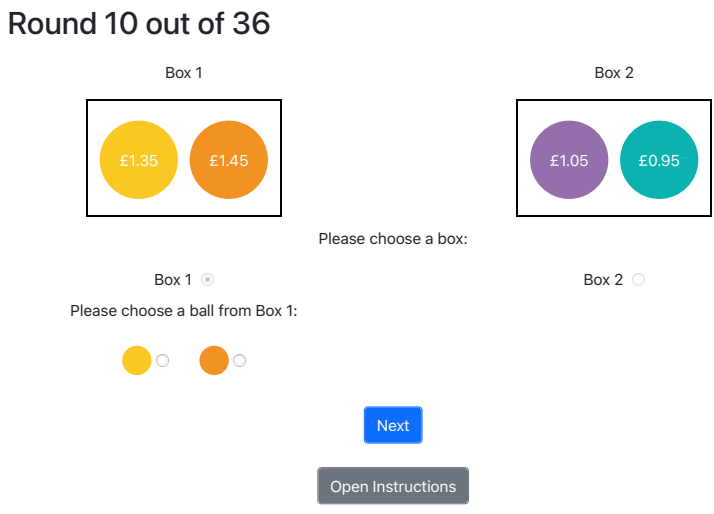

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**Choices**:

WYSIWYG payment + £2.55 completion

225 participants.  Avg pay £4.35;  Median duration 7min

# Experimental Design: Beliefs

**Experimental framework to study off-path beliefs**

Elicit beliefs about choices at terminal info set

Compare to beliefs about choices same info set when on-path vs off-path

1. Elicited choices first (experiment 1)

2. Elicit beliefs (experiment 2)

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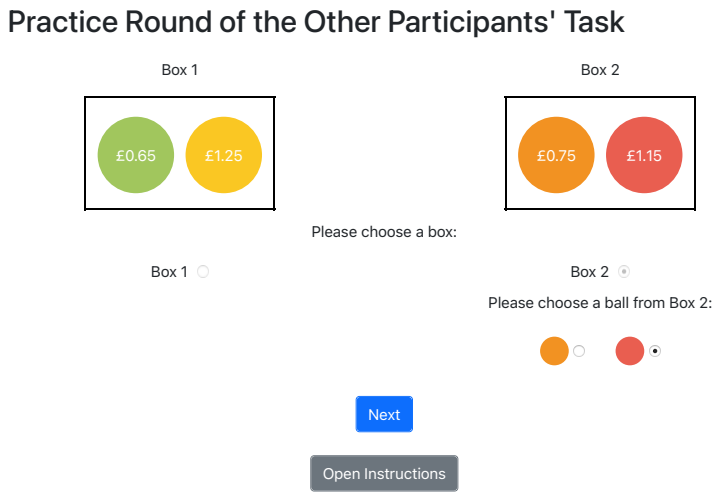
   Explain other's task (practice)

   Elicit beliefs

# Experimental Design: Beliefs



## Practice Round of the Other Participants' Task

Box 1


Box 2

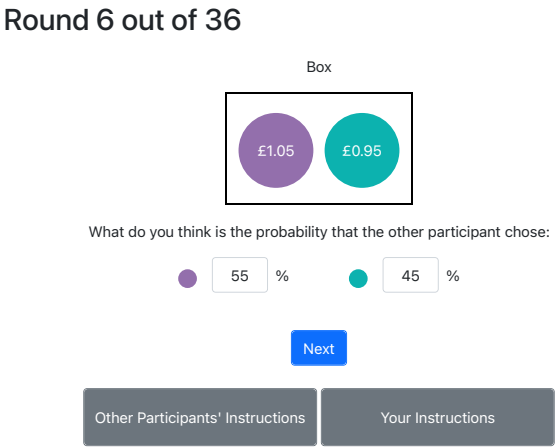

In the practice round, you chose Box 2 and the ball.

If this were a paid round and the other participant had made this choice, they would be paid a bonus of £1.15.

Click 'Next' to see your instructions for this study.

Next

# Experimental Design: Beliefs



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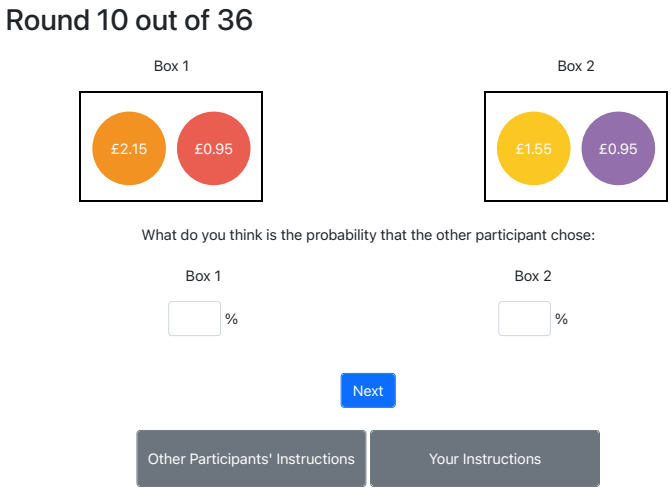
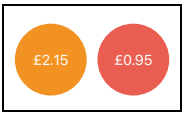


# Experimental Design: Beliefs

## Round 10 out of 36

Box 1




You predicted the following probabilities of the other participant choosing Box 1:

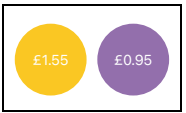
5.0 %

**Suppose you've been matched with one of the other participants who chose Box 1.**

What do you think is the probability that the other participant chose:

80 % 20 %

Box 2




You predicted the following probabilities of the other participant choosing Box 2:

95.0 %

**Suppose you've been matched with one of the other participants who chose Box 2.**

What do you think is the probability that the other participant chose:

50 % 50 %

Next

Other Participants' Instructions | Your Instructions

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**Two parts**

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1 round randomly selected for payment

**Beliefs**:

BSR £3 vs 0 + £3 completion

225 participants.  Avg pay £5.70;  Median duration 17min

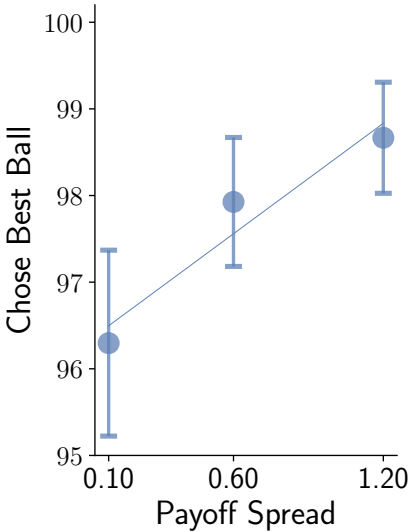
# Choice and Beliefs in One-Shot Settings

# More Indifferent $\Longrightarrow$ More Mistakes

+ Spread

$\Longrightarrow$ Less indifferent

$\Longrightarrow$ Fewer mistakes

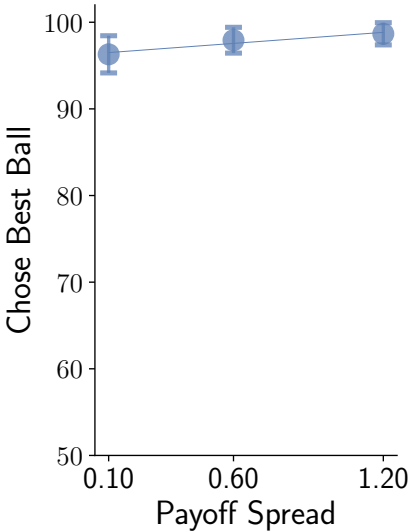



(Figs: 95% CI with clustered se's at participant level)

# More Indifferent $\Longrightarrow$ More Mistakes

But do note the *y*-axis...

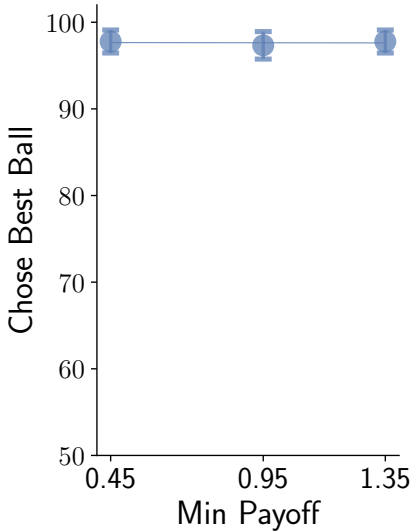
Basically very few mistakes anyway (<2.5%)



# Level Shifts Don’t Matter

No effect of min

i.e., adding a constant has no effect

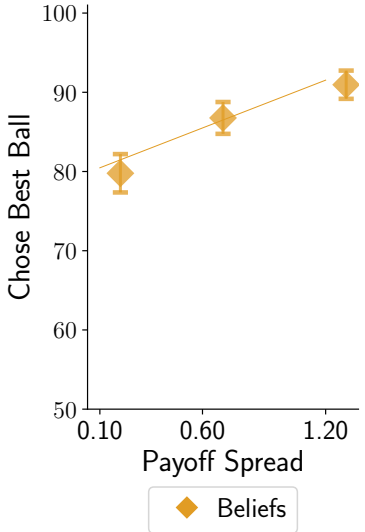


# More Indifferent $\Longrightarrow$ Believe More Mistakes

+ Spread

$\Longrightarrow$ Less indifferent

$\Longrightarrow$ Believe in fewer mistakes



# Believe that Level Shifts Don't Matter

Also no effect of min

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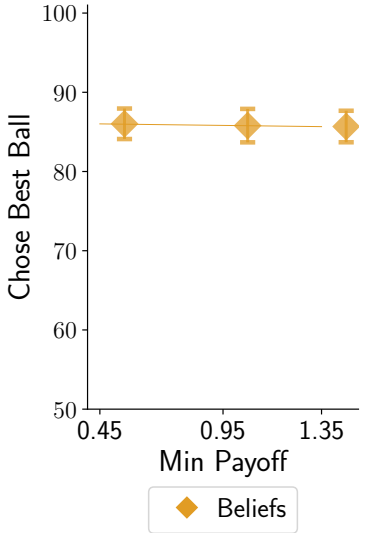
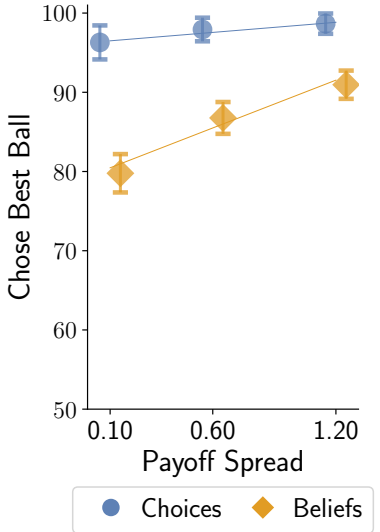


# Beliefs vs Choices

Overestimate others' mistakes

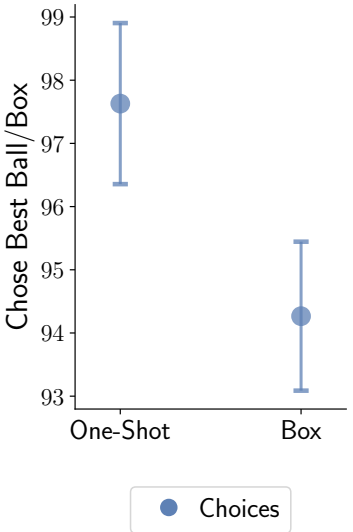
Beliefs react more to fundamentals than choices



# The Off-Path

# Is there an Off-Path?

Barely: on avg, chose off-path <6%



# What Makes Off-Path Off-Path?

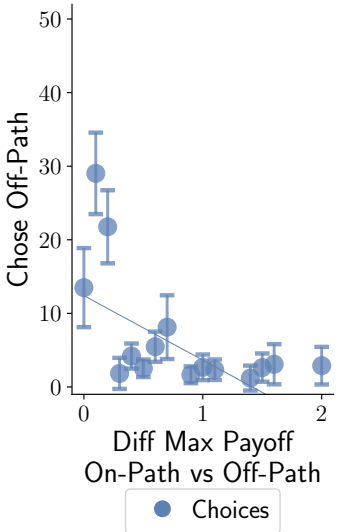
Larger difference in max payoff

$\Longrightarrow$ Off-Path worse

$\Longrightarrow$ Less indifferent

$\Longrightarrow$ Fewer mistakes/off-path choices

(On avg off-path <6%)



# What Makes Off-Path Off-Path?

Smaller difference in min payoff

$\Longrightarrow$ Off-Path relatively safer

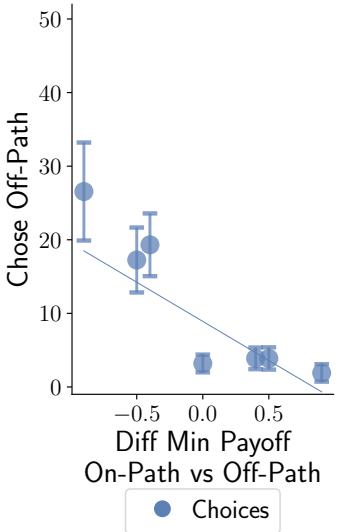
$\Longrightarrow$ Fewer mistakes/off-path choices

Difference in min negative

$\Longrightarrow$ Off-Path min > On-Path min

(by definition, Off-Path max < On-Path max)

'Trade-off' gain vs risk? (temptation/self-control problem?)



# Do People Actually Believe There's an Off-Path?

Yes. (i.e., much more than I thought they/I would)

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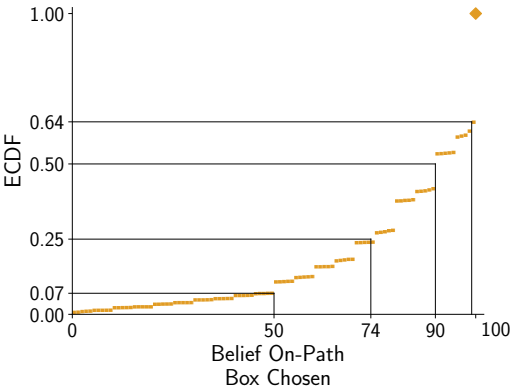
36% of instances: on-path wp 1

>50% of instances: on-path wp $\geq$ 90%



# What Determines Beliefs in the Off-Path?

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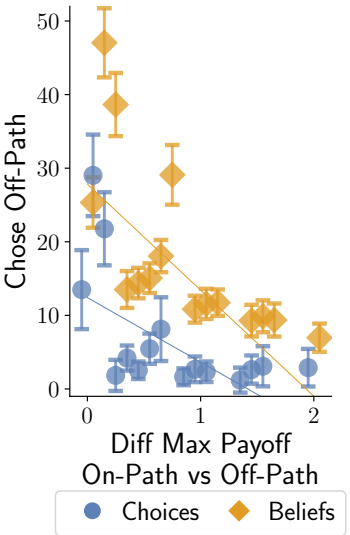

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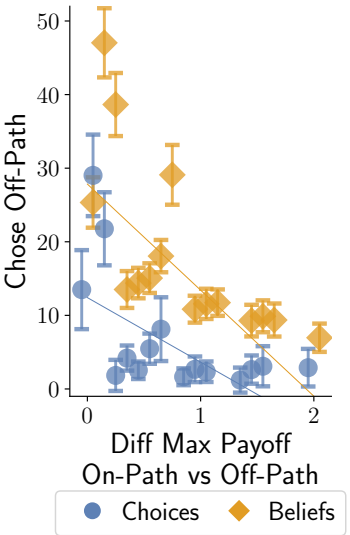
Note: almost looks like level shift

Beliefs react more to fundamentals than choices



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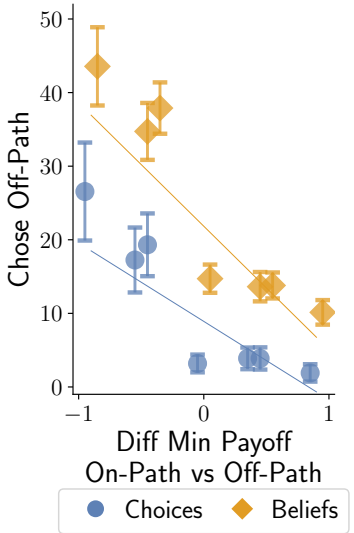
Believe in others' sophistication regarding their own self-control?



# On the Off-Path

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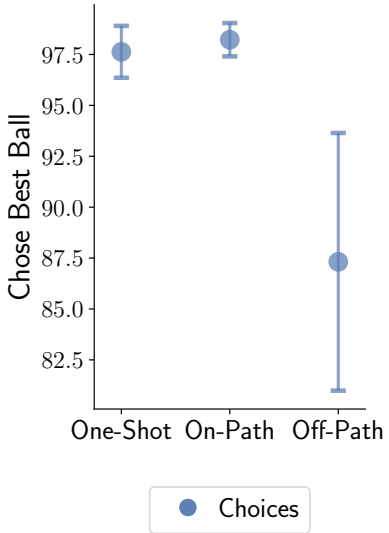
Greater likelihood of mistake following Off-Path

- On-Path mistakes even less frequent
- Off-Path mistakes more frequent



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Also: Mistakes in One Shot predictive of choosing Off-Path:

- Freq (Choosing Off-Path | No Mistake in One-Shot) = 4%
- Freq (Choosing Off-Path | Made Mistake in One-Shot) = 24%
- Corr (Nr. Choices Off-Path, Nr. Made Mistake in One-Shot) = .6

# Beliefs On- and Off-Path

Believe in greater likelihood of mistake following Off-Path

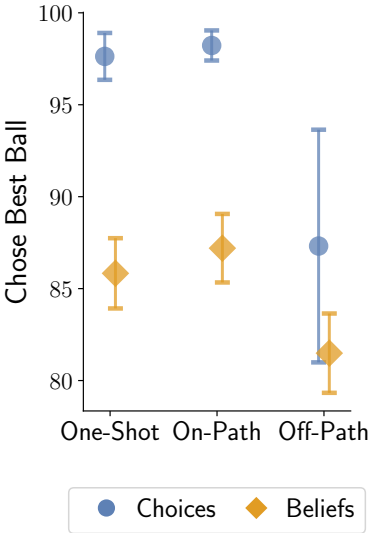
On-Path infer mistakes slightly less frequent

Off-Path infer mistakes somewhat more frequent

Recall: not necessarily the case

irrelevant mistakes, more mistakes from subsequent choices

Interestingly: now beliefs seem to react *less* than choices



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log-odds Belief Mistake | Off-Path = $\beta_1$ log-odds Belief Mistake *in One-Shot* + $\beta_2$ log-likelihood Off-Path

Estimates: base-rate neglect $\beta_1 \approx .9$ and severe underinference $\beta_2 \approx .2$

Note: under-reaction of beliefs specifically about off-path correlation, not general feature:

e.g., beliefs do react much more than choices to differences in payoffs

# True Equilibrium Believers

# Who *Truly* Believes Off-Path is Off-Path?

Surprisingly many people!

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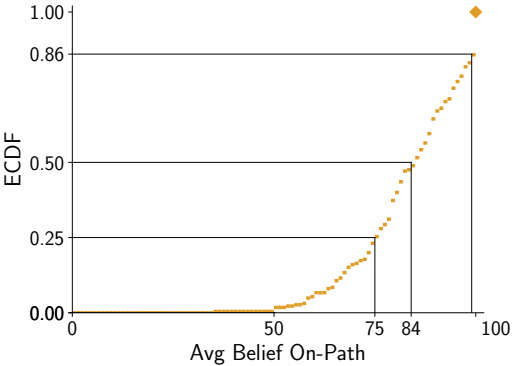
Surprisingly many people!

14% of true believers:

*always* assign prob 1 to on-path

Median participant:

on avg believes on-path chosen 84% of the time

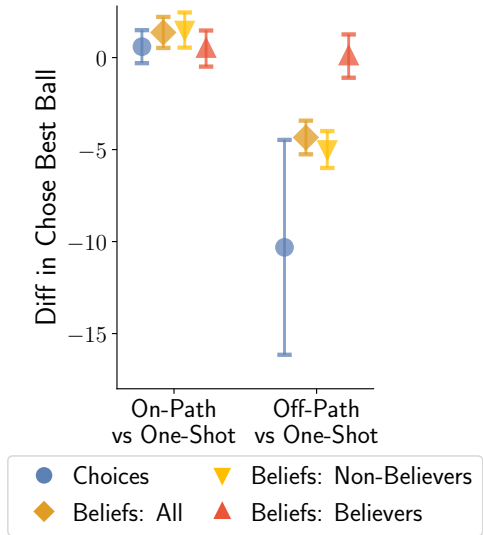


# Updating from Zero Probability Event

"Equilibrium believers": believe others play on-path and behave coherently, both on and off path

Always assign prob. 1 to on-path and almost always optimal behaviour, both on and off-path

Conditional on on- or off-path event, true believers don't revise their model
(from diff-in-diff, also via Grether regression)



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(1) overpredict noise,

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Most people

(1) overpredict noise,

(2) underinfer from off-path behaviour, neglecting correlation in mistakes

True believers in on-path behaviour are true believers:

believe mistakes never happen, and, when they do, keep believing they just won't happen again

Heterogeneity: apart from faster people inferring slightly more, other dimensions not significant (age, uni, etc)

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**THANK YOU!**

# Overview

1. Experimental Design
   - Choices
   - Beliefs

2. Choice and Beliefs in One-Shot Settings
   - Choice Mistakes
   - Beliefs about Mistakes

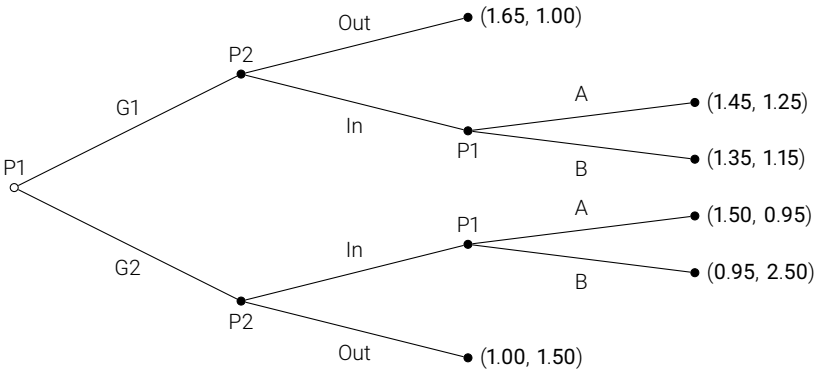
3. The Off-Path
   - Choosing the Off-Path
   - Believing in the Off-Path

4. On the Off-Path

5. True Equilibrium Believers

6. Take-aways

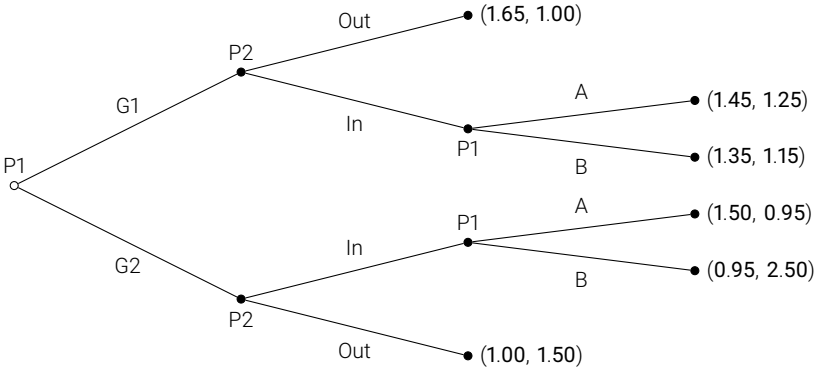
# Example




P1 prefers A to B in both G1 and G2. P2 then prefers In in G1 and Out in G2. P1 then prefers G1.

Unique SPNE (G1, A, A) and (In, Out); indeed, *unique NE outcome* G1-In-A

# Example




P1 prefers A to B in both G1 and G2. P2 then prefers In in G1 and Out in G2. P1 then prefers G1.

Unique SPNE (G1, A, A) and (In, Out); indeed, *unique NE outcome* G1-In-A

But... if P2 sees G2 (off-path move), what to think about follow on moves? In or Out?

Depends on beliefs about what P1 does after... Stick to believing P1 chooses A despite mistake?