

# Sequential Sampling Equilibrium

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### Sequential Sampling

Relates choice and time. (Ratcliff '78; Fudenberg, Strack, & Strzalecki '18)

Dynamic costly information acquisition.

Past experiences, memories, reasoning. ( Gold & Shadlen '07, Shadlen & Shohamy '16 )

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Higher stakes, slower choices, more sophisticated actions and beliefs.

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## **Sequential Sampling** *Equilibrium*

Sampling: Players form beliefs by acquiring signals about others' behaviour.

Sequential: Information acquisition is sequential and costly.

Strategic uncertainty drives information acquisition.

Equilibrium: signals informative of others' behaviour.

## Sequential Sampling Equilibrium

Disciplined model with endogenous distrib. choices, beliefs, and decision times.

General comparative statics results.

Generate stochastic choice via info acquisition,  
not relying on indifference or mistakes.

## Behavioural Implications

Rationalise patterns of response times in games.

Payoffs  $\rightarrow$  decision time  $\rightarrow$  sophistication of play.

Time-revealed preference intensity.

Explain well-known deviations from Nash eqm and evidence on beliefs  
via sequential sampling.

## A Rationale for Other Solution Concepts

Recover other equilibria as limit case, (Bayesian) Nash equilibrium, ABEE.

# Overview

1. Sequential Sampling
2. Equilibrium
3. Absolute Incentives, Rationalisability, and Decision Time
4. Sequential Sampling, Choices, and Comparative Statics
5. Preference Intensity and Decision Time
6. Relation to Nash Equilibrium
7. Extensions
8. Conclusion

# Setup

**Game**  $\Gamma = \langle I, A, u \rangle$

Players  $I$ , Player  $i$ 's Actions  $A_i$ , finite, Payoffs  $u_i : A \rightarrow \mathbb{R}$ , Opponents  $-i$ .

Randomisation  $\sigma_i \in \Delta(A_i)$ ;  $u_i(\sigma_i, \sigma_{-i}) = \mathbb{E}_{\sigma}[u_i(a_i, a_{-i})]$

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Uniform **beliefs** about  $\sigma_{-i}$ ,  $\mu_j$ .

Players are uncertain about opponents' action distribution  $\sigma_{-j}$ .

Laplacian principle of insufficient reason.

## Setup

Before choosing, Player  $i$  can **acquire information** about unknown prob. distrib.  $\sigma_{-i}$ .

Each period  $t$ , get signal informative signal  $y_{i,t}$  about  $\sigma_{-i}$  cost  $c_j$ .

Sample up to time  $t$ ,  $y_i^t := (y_{i,1}, \dots, y_{i,t})$ .

Response time due to costly reasoning: *acquiring data, asking friends, recalling past experiences.*

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Simplifying assumptions (can be relaxed):

uniform prior,  $y_{i,t} \sim \sigma_{-i}$ , constant cost of time, game of complete information.

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Player  $i$ 's **optimal stopping** problem:

$$V_i(\boldsymbol{\mu}_i) := \sup_{t_i \in \mathbb{T}_i} \mathbb{E}_{\boldsymbol{\mu}_i} \left[ \max_{a_i} \mathbb{E}_{\boldsymbol{\mu}_i} [u_i(a_i, \boldsymbol{\sigma}_{-i}) \mid y_i^{t_i}] - c_i \cdot t_i \right]$$

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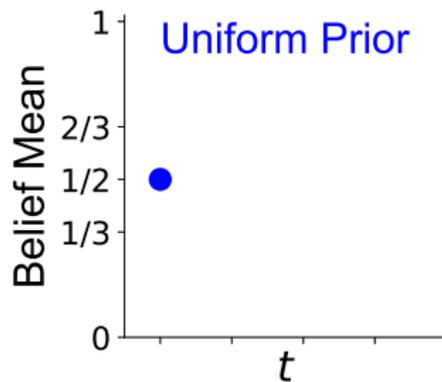
**Optimal stopping time**  $\tau_i$

$$\tau_i = \inf \left\{ t \geq 0 \mid V_i(\mu_i \mid y_i^t) = \mathbb{E}_{\mu_i}[u_i(a_i, \sigma_{-i}) \mid y_i^t] \right\}$$

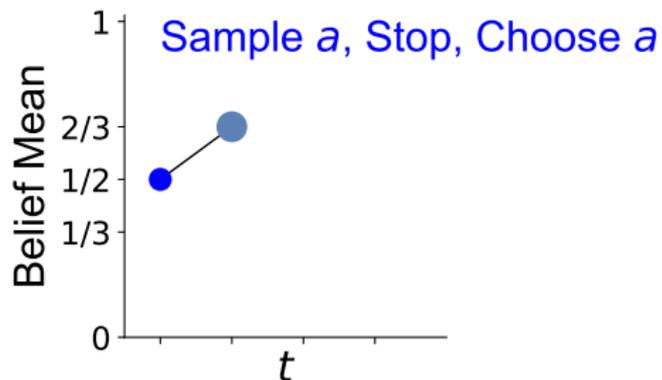
## Optimal Sequential Sampling: An Example

2x2 Game,  $A_i = A_j = \{a, b\}$ .

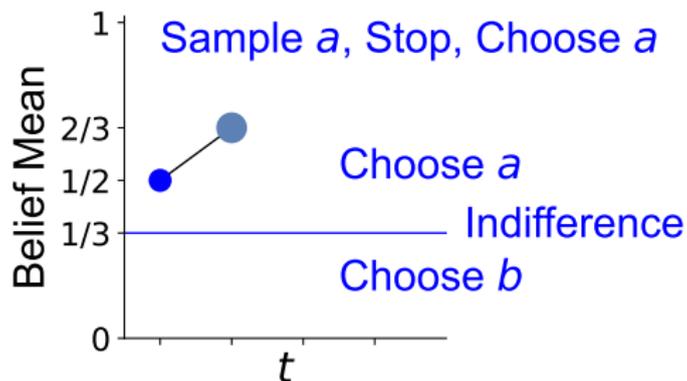
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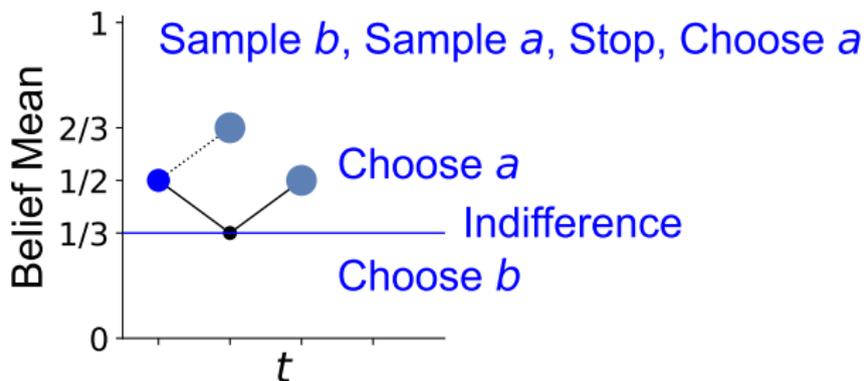
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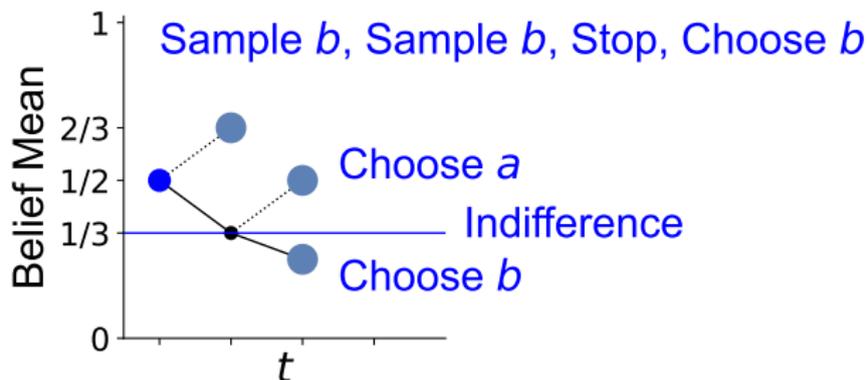
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# Overview

1. Sequential Sampling
2. Equilibrium
  - Defining an Equilibrium
  - Related Models
  - Existence
3. Absolute Incentives, Rationalisability, and Decision Time
4. Sequential Sampling, Choices, and Comparative Statics
5. Preference Intensity and Decision Time
6. Relation to Nash Equilibrium
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# Defining an Equilibrium

## Closing the model

Sampling is informative: induced by opponents' distribution of actions  $\sigma_{-j}$ .

Randomness in sampling: induces a distribution of actions for player  $i$ .

Equilibrium as stationary condition / fixed point.

## Stochastic Choice

$\Delta(\text{opponents' actions}) \rightarrow \Delta(\text{sample paths}) \rightarrow \Delta(\text{beliefs upon stopping}) \rightarrow \Delta(\text{own actions})$ .

$b_i$  maps from opponents' action distribution to Player  $i$ 's.

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Player  $i$ 's **set of stopping sample paths**  $\mathcal{Y}_i^{\tau_i}$  according to optimal stopping time  $\tau_i$ .

$\mathcal{Y}_i^{\tau_i}$  depends only on payoffs and costs; not on  $\sigma_{-i}$ .

**Selection of optimal choices** for any given beliefs:  $a_i^*(\mu_i) \in \arg \max_{a_i \in A_i} \mathbb{E}_{\mu_i} [u_i(a_i, \sigma_{-i})]$ .

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# Defining an Equilibrium

## Definition

A **sequential sampling equilibrium** is a profile of action distributions  $\sigma$  such that, for every  $i$ ,  $\sigma_i = b_i(\sigma_{-i})$ .

Equilibrium as a consistency condition on action distributions:

- (i) Each player  $i$  samples from opponents' distribution of actions  $\sigma_{-i}$ ;
- (ii) Sequential sampling induces a distribution of actions for player  $i$ .

Equilibrium  $\sigma_{-i}$  and optimal sequential sampling

$\implies$  pin-down equilibrium joint distrib. choices, beliefs, and time  $(a_i, \tau_i)$ .

Decision time determined in equilibrium ( $\neq$  subjective expected info costs).

**Sequential Sampling and Response Time.** Ratcliff ('78), Fudenberg, Strack, & Strzalecki ('18), Ke & Villas-Boas ('19), Cerreia-Vioglio et al. ('22), Alós-Ferrer, Fehr, & Netzer ('22), Gonçalves WP.

**Sampling in Games/Costly Information Acquisition in Games.** Osborne & Rubinstein ('98, '03), Salant & Cherry ('20); Yang ('15), Denti ('22), Hébert & La'O ('22); Alaoui & Penta ('16, '22).

**Learning in Games.** Battigalli ('87), Fudenberg & Levine ('93), Fudenberg & Kreps ('93), Esponda & Pouzo ('16), Battigalli et al. ('15), Fudenberg, Lanzani, & Strack ('21, '24).

### **This paper:**

**sequential sampling:** driven by strategic uncertainty, rationalise response time;

**equilibrium:** joint distribution choices, beliefs, & time determined endogenously.

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Any finite game has a sequential sampling equilibrium.

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## Proof intuition

- $b_i(\sigma_{-i}) \in \Delta(A_i) + \text{Continuity } b_i + \text{Brouwer FPThm} = \text{Existence}$
- $b_i(\sigma_{-i}) \in \Delta(A_i) + \text{Continuity } b_i$  iff  $\tau_i$  is finite wp1 wrt  $\sigma_{-i}$

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- Full support guarantees *stopping time bounded*

Beliefs concentrate around empirical distrib. at uniform rate.

Decreasing value to sampling further.

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- Full support guarantees *stopping time bounded*  
Beliefs concentrate around empirical distrib. at uniform rate.  
Decreasing value to sampling further.

Result generalises: non-constant sampling costs, general information structures.

Finite upper bound useful for computational tractability.

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- 3. Absolute Incentives, Rationalisability, and Decision Time**
4. Sequential Sampling, Choices, and Comparative Statics
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# Absolute Incentives, Rationalisability, and Decision Time

## Rationalisability

$a_i$  is 1-rationalisable if there is  $\sigma_{-i}$  s.t. such that  $a_i$  is a best-response to  $\sigma_{-i}$

$a_i$  is  $(k + 1)$ -rationalisable if there is  $\sigma_{-i}$  on  $k$ -rationalisable actions s.t. such that  $a_i$  is a best-response to  $\sigma_{-i}$

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## Evidence for Dominance-Solvable games

Scaling up payoffs  $\rightarrow$  longer decision time and action sophistication.

Longer decision time associated with

- more sophisticated play (higher-order rationalisability);
- beliefs ascribing greater sophistication to opponents.

(Rubinstein '07, '16; Esteban-Casanelles & Gonçalves WP; Alós-Ferrer & Buckenmaier '21)

### Theorem

Let  $\Gamma_\lambda := \langle I, A, \lambda u \rangle$  and fix  $c$ . For any  $k$ , there  $\lambda > 0$  such that, in any sequential sampling equilibrium of  $\Gamma_\lambda$ , only  $k$ -rationalisable actions are played with positive probability.

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Relation between sampling costs and sophistication of equilibrium choices:  
e.g., level- $k$  actions and sampling costs in dominance solvable games.

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## Proof intuition

- (i) If assign high enough prob. to others only choosing  $(k - 1)$ -rationalisable actions, then will choose  $k$ -rationalisable action.
- (ii) If everyone else is indeed only choosing  $(k - 1)$ -rationalisable actions, need only to sample enough to believe this is the case with high prob.
- (iii) If player  $i$  has no weakly dominant action, then for every  $\underline{T}_i$ , if sampling cost is low enough, Player  $i$  samples at least  $\underline{T}_i$  observations (lower bound on stopping time).
- (iv) Scaling up payoffs equiv. to scaling down costs.

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  - Equilibrium Comparative Statics: Applications
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# General Comparative Statics for Optimal Stopping Problems

## General Comparative Statics on Optimal Stopping

Individual decision problem:

Treat  $\sigma_{-j} \equiv \theta \in [0, 1]$  as fixed exogenous unknown parameter,  $u_j(a_j, \theta)$ .

Focus on binary action problems:  $A_j = \{a, b\}$ ;

WLOG assume  $\theta \mapsto u_j(a, \theta) - u_j(b, \theta)$  increasing.

Increasing payoffs to action  $a$ :

$$\tilde{u}_j(a, \theta) := u_j(a, \theta) + f(\theta), f \geq 0.$$

# General Comparative Statics for Optimal Stopping Problems

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(i) Expands stopping region for  $a$  and shrinks that for  $b$ ;  
need less (more) information to be convinced to stop and choose  $a$  ( $b$ ).

(Gonçalves, Nunnari, & Zarate-Pina WP)

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- (iii) is smooth and increasing  $\theta$ .

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- (ii) Prior already more favourable to choosing action  $a$ ;
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Results extend to multiple actions and multidimensional  $\theta$ , general information structures, general prior, general sampling costs.

Importantly: results for *objective* measure  $\mathbb{P}_{\theta}$  (also true for subjective measure  $\mathbb{P}_{\mu_i}$ ).

*Not* generally true for flexible information acquisition (Pease & Whitmeyer WP).

## Application: Uniqueness

### Corollary

Any  $2 \times 2$  game with unique Nash eqm generically\* has unique sequential sampling eqm.

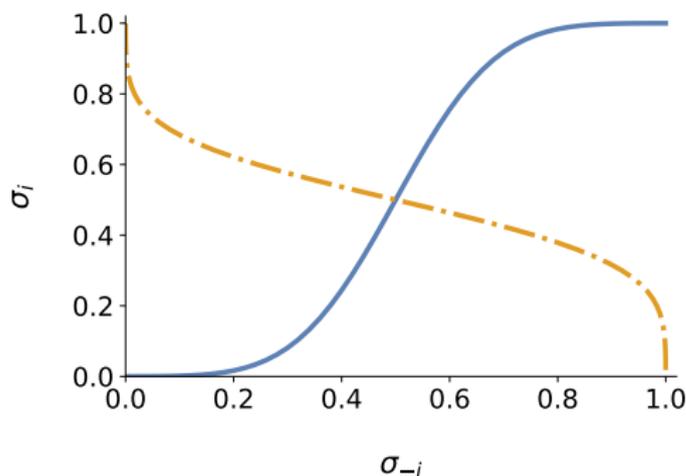
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## Application: Equilibrium Comparative Statics

		Clasher	
		<i>a</i>	<i>b</i>
Matcher	<i>a</i>	$\delta_M, 0$	$0, 1$
	<i>b</i>	$0, 1$	$1, 0$

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	<i>b</i>	$0, 1$	$1, 0$

### Corollary

Increasing Matcher's payoffs to action *a*,

- (i) increases  $\mathbb{P}\{\text{Matcher choosing action } a\}$  and  $\mathbb{P}\{\text{Clasher choosing action } b\}$ ;

- (i) Rationalise commonly observed deviation from Nash equilibrium via *info acquisition*.

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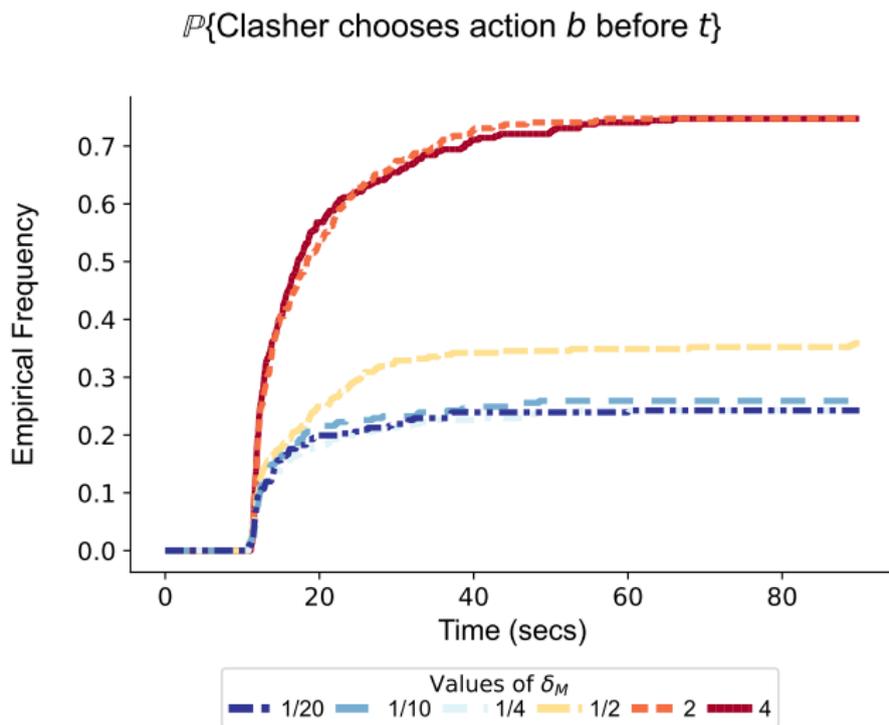
(ii) Uncover new (equilibrium) relation between time and choice:

Increasing **Matcher's** payoffs to **action a** leads

**Clasher** to choose **action b** more often and *faster*  
and **action a** less often and *slower*.

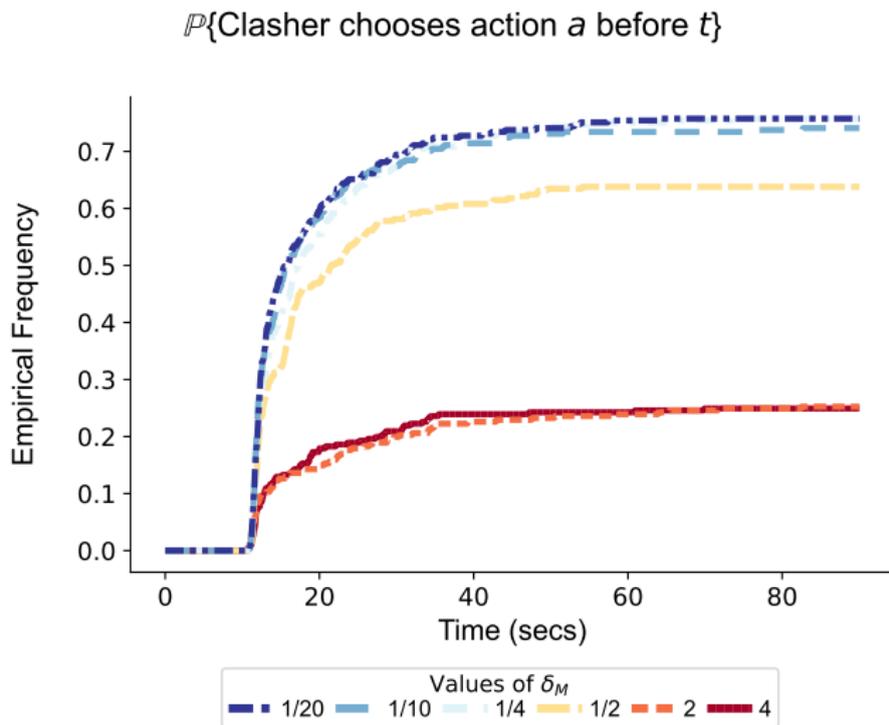
This isn't because **Clasher's** payoffs change, but because **Matcher's** eqm prob. does!

# Application: Equilibrium Comparative Statics: Evidence (1)



Data from Friedman & Ward (WP)

## Application: Equilibrium Comparative Statics: Evidence (2)



Data from Friedman & Ward (WP)

# Overview

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5. Preference Intensity and Decision Time
  - Sequential Sampling and Beliefs
  - Application: Equilibrium and Beliefs
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# Sequential Sampling and Beliefs

Own incentives affect beliefs holding fixed opponents. (Esteban-Casanelles & Gonçalves WP)

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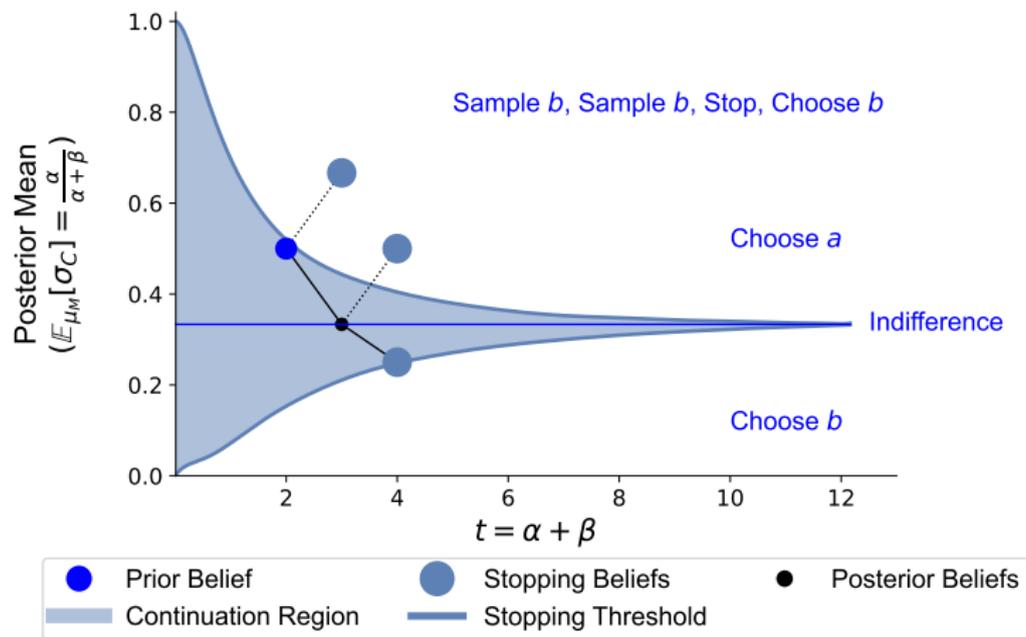
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$\mathbb{E}$  Eqm beliefs upon stopping  $\neq$  opponents' gameplay action distrib.

Bias toward prior; depends on payoffs and decision time.

# Sequential Sampling and Beliefs



### Proposition

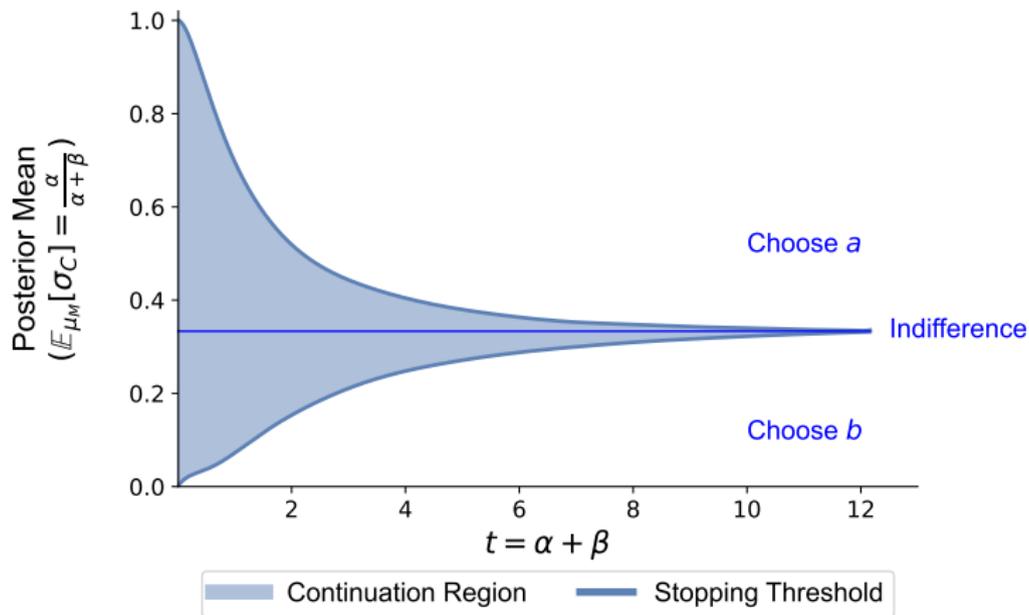
There are  $\underline{\sigma}_{-i}(t), \bar{\sigma}_{-i}(t)$  such that

- (i) player  $i$  keeps sampling if and only if posterior mean  $\in (\underline{\sigma}_{-i}(t), \bar{\sigma}_{-i}(t))$
- (ii)  $\bar{\sigma}_{-i}(t)$  is decreasing and  $\underline{\sigma}_{-i}$  is increasing in  $t$   
and converge to player  $i$ 's indifference point  $\check{\sigma}_{-i} : u_i(a, \check{\sigma}_{-i}) = u_i(b, \check{\sigma}_{-i})$ .

Continuation region **shrinks** to indifference point.

Reminiscent of Fudenberg, Strack, & Strzalecki ('18) but with correlation in payoffs.

# Sequential Sampling and Beliefs



## Time-Revealed Preference Intensity

Slower decisions  $\implies$  Player closer to indifferent.

Evidence for decision-making (Konovalov & Krajbich '19; Alós-Ferrer, Fehr, & Netzer '22)  
and in strategic settings (Schotter & Trevino '21; Friedman & Ward WP).

## Applications: Comparative Statics in Equilibrium Beliefs

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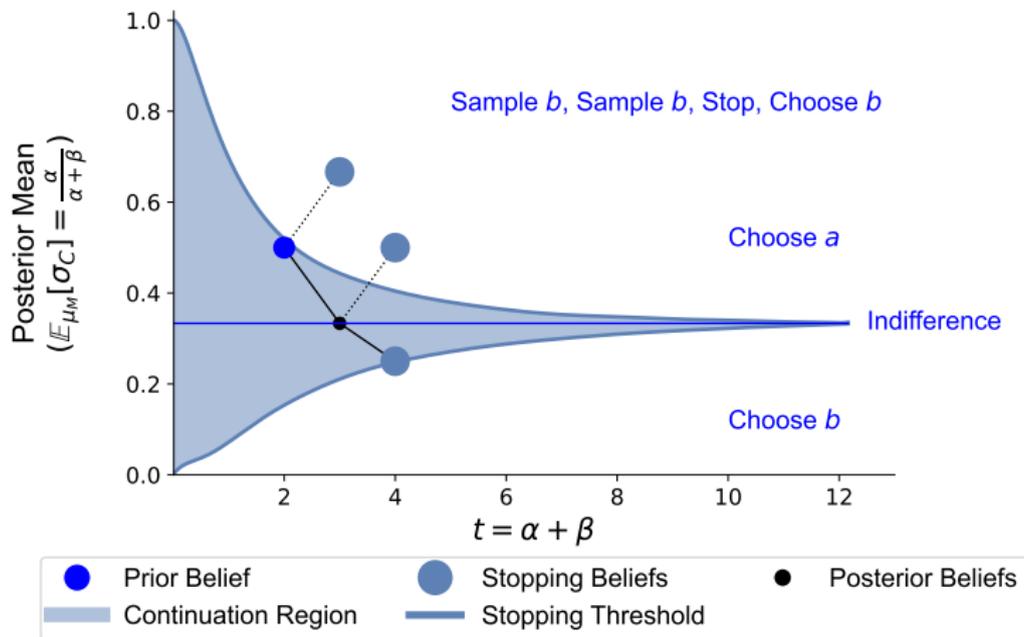
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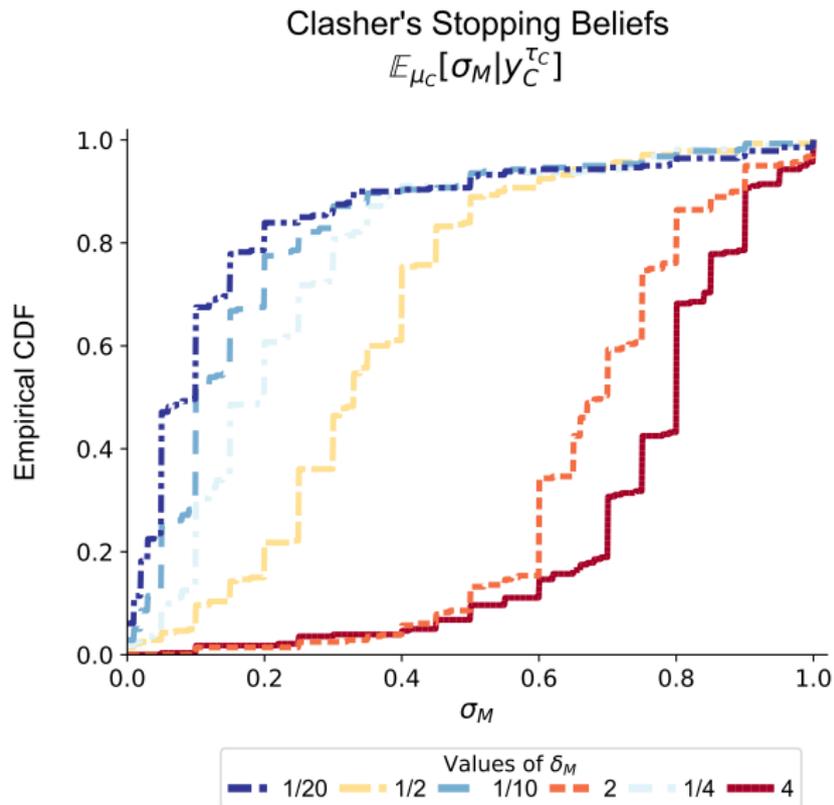
- (i) increases Clasher eqm. posterior mean in FOSD sense.

Follows from eqm comparative statics + collapsing boundaries.

# Applications: Comparative Statics in Equilibrium Beliefs



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Data from Friedman & Ward (WP)

## Applications: Comparative Statics in Equilibrium Beliefs (Bis)

Strategic substitutes: market entry, congestion, queueing.

Continuum of players  $i \in [0, 1]$ .  $\sigma$ : prob. choose risky action.

Risky action:  $u_i(r, \sigma_{-i}) = r(1 - \bar{\sigma})$ . Safe action:  $u_i(s, \sigma_{-i}) = s_i$ ,  $s_i \sim U(0, 1)$ ;  $r > 0$ .

Indifference:  $\bar{\sigma}_{-i} = 1 - s_i/r$ .

Essentially unique sym. BNE:  $\sigma_i(s_i) := \mathbf{1}\{s_i < r/(r+1)\}$ ;  $\bar{\sigma}^{NE} = r/(r+1)$ .

### Proposition

- (i) There is an essentially unique symmetric sequential sampling eqm.  
Furthermore,  $r \geq (\leq) 1 \implies \bar{\sigma}^{SSE} \geq (\leq) \bar{\sigma}^{NE}$ .
- (ii) With higher  $s_i$ ,  $\mathbb{P}(\text{player } i \text{ chooses } a \text{ before } t)$  decreases,  
 $\mathbb{P}(\text{player } i \text{ chooses } b \text{ before } t)$  increases,  
and stopping posterior beliefs  $\mathbb{E}_{\mu_i}[\sigma_{-i}|y_i^{t_i}]$  decrease in FOSD sense.

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$\mathbb{P}(\text{Choose risky}) = \mathbb{P}(s_i < r \mathbb{E}_{\mu_i}[1 - \bar{\sigma} | y_i^{t_i}])$ .

Bias toward prior:  $\mathbb{P}(s_i < r \mathbb{E}_{\mu_i}[1 - \bar{\sigma}]) = r/2 \geq r/(r+1) = \bar{\sigma}^{NE} \iff r \geq 1$ .

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## Relation to Nash Equilibrium

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Cost decreases  $\rightarrow$  Players sample more  $\rightarrow$  Law large numbers  
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**Restore convergence** using concentration inequalities + martingale theory

## Nash Equilibria as Limit Points

### Theorem

Let  $c_n \rightarrow 0$ . For any sequence  $\{\sigma_n\}$  of sequential sampling eqa, its limit points are Nash equilibria.

### Proof Intuition

- (i)  $\downarrow c_j$ , sample at least  $\uparrow T_j$  obs.
  - (ii) Posterior belief concentrates uniformly around empirical mean  $\bar{y}_i^{T_j}$ .
  - (iii)  $\|\bar{y}_i^{T_j} - \sigma_{-j}\|$  is supermartingale wrt  $\sigma_{-j}$ .
  - (iv) Wald's identity:  $\mathbb{E}_{\sigma_{-j}}[\|\bar{y}_i^{T_j} - \sigma_{-j}\|^2] \leq \mathbb{E}_{\sigma_{-j}}[|A_i|/\tau_j] \leq |A_i|/T_j$ ,  
hence  $\mathbb{E}_{\sigma_{-j}}[\|\bar{y}_i^{T_j} - \sigma_{-j}\|] \leq \sqrt{|A_i|/T_j}$ .
- $\implies$  Players learn true distribution as costs decrease.

(Selection)

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Players have types (finite type space).

Samples include players' type profiles; e.g., learn from behaviour in similar games.

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## More General Information Structures

Garbled observations: noisy recollections, payoff sampling, info imperfectly distinguishes actions/types/players.

Special case: info structure is partitional.

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## More General Information Structures

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Convergence to analogy-based expectations eqm.

Can accommodate: non-uniform priors, non-constant cost.

Discuss: steady-state foundation, Nash eqm selection, myopic info acquisition, misspecified priors, estimation.

# Recap

## Sequential Sampling Equilibrium

Disciplined model with endogenous distrib. choices, beliefs, and decision times.

General comparative statics results.

Generate stochastic choice via info acquisition

not relying on indifference or mistakes.

## Behavioural Implications

Rationalise patterns of response times in games

Payoffs  $\rightarrow$  decision time  $\rightarrow$  sophistication of play

Time-revealed preference intensity.

Explain well-known deviations from Nash eqm and evidence on beliefs via sequential sampling.

## A Rationale for Other Solution Concepts

Recover other equilibria as limit case, (Bayesian) Nash equilibrium, ABEE.

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**Thank you!**

# Sequential Sampling Equilibrium

Duarte Gonçalves

University College London

Universitat Autònoma de Barcelona

19 March 2026

## Conditions for Existence

Full support sufficient, not necessary: e.g. degenerate priors, no sampling

**Existence not assured** for general misspecified priors

Players **believe they will stop** sampling with prob. 1 (wrt. their prior)

Need players to **actually stop** sampling with prob. 1 (wrt. true distrib. samples)

(Back)

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Need players to **actually stop** sampling with prob. 1 (wrt. true distrib. samples)

But may never stop sampling under opponents' true distrib. actions

Precludes existence of an equilibrium (Example)

(Back)

## Misspecified and Non-Existence: Example

		Player 2		
		<i>L</i>	<i>C</i>	<i>R</i>
Player 1	<i>U</i>	1,0	0,1	0,0
	<i>D</i>	0,0	0,1	1,0

*C* strictly dominant action for Player 2

(Back)

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$\mu_1$  assigns equal probability to  $\sigma'_2 = (1 - 3\varepsilon, 2\varepsilon, \varepsilon)$  and to  $\sigma''_2 = (\varepsilon, 2\varepsilon, 1 - 3\varepsilon)$

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(Back)

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When Player 1 observes C, prior = posterior

Value of one additional sample to Player 1 strictly positive under prior  $\mu_1$

If sampling cost is low enough, Player 1 wants to sample at least once

Player 1 always gets C  $\implies$  prior always = posterior

$\implies$  Player 1 never stops sampling

(Back)

# Sampling from Past Data

Periods  $n = 1, 2, \dots$

Extended game  $G = \langle \Gamma, \mu, c \rangle$ ; full-support priors

Each period: agents each period randomly matched to play game  $\Gamma$

For simplicity, assume measure 1 of agents for each role (myopic behaviour)

Fix priors  $\mu$  and sampling costs  $c$

Agents sample from past realisations

fictitious play with costly sequential sampling

(Back)

## Sampling from Past Data

Players sequentially sample observations from dataset with past actions

Arbitrary starting point  $\sigma_0$ .

Accumulated past actions at  $n$  distrib.  $\sigma^{n-1}$ .

→ Distribution of actions in period  $n$  given by  $b(\sigma^{n-1}) := (b_i(\sigma_{-i}^{n-1}))_{i \in I}$ .

→ Accumulated past actions distributed according to

$$\sigma^n = \frac{n}{n+1} \sigma^{n-1} + \frac{1}{n+1} b(\sigma^{n-1}).$$

(Back)

# Sequential Sampling Equilibria as Steady States

## Theorem

(1)  $\sigma$  is the limit of  $\{\sigma^n\}_n$  for some  $\sigma^0$  if and only if (2)  $\sigma$  is a sequential sampling eqm.

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## Proof Sketch

(2) $\implies$ (1):  $\sigma^0$  seq. sampling eqm.  $\sigma^0 = b(\sigma^0) = \sigma$ .

(1) $\implies$ (2): Crux is showing  $b(\sigma^n) \rightarrow \sigma$ .

(Back)

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(Back)

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- Continuity  $b$  implies  $\sigma^n \rightarrow \sigma \implies b(\sigma^n) \rightarrow b(\sigma)$ .
- Cesàro mean converges:  $\bar{\sigma}^{n-1} := \frac{1}{n} \sum_{\ell=0}^{n-1} b(\sigma^\ell) \rightarrow b(\sigma)$ .

(Back)

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- Cesàro mean converges:  $\bar{\sigma}^{n-1} := \frac{1}{n} \sum_{\ell=0}^{n-1} b(\sigma^\ell) \rightarrow b(\sigma)$ .
- Result follows:  $\sigma^n = \frac{1}{n+1} \sigma^0 + \frac{n}{n+1} \bar{\sigma}^{n-1} \rightarrow b(\sigma)$  and  $\sigma^n \rightarrow \sigma$ .

(Back)

# Sequential Sampling Equilibria as Steady States

## Theorem

(1)  $\sigma$  is the limit of  $\{\sigma^n\}_n$  for some  $\sigma^0$  if and only if (2)  $\sigma$  is a sequential sampling eqm.

Result generalises:

- (i) to sampling from own past experiences (instead of all past data);
- (ii) to other data accumulating processes,

e.g., weighting past data exponentially:

$$\sigma^n = \alpha \cdot \sigma^{n-1} + (1 - \alpha) \cdot b(\sigma^{n-1}), \quad \alpha \in (0, 1);$$

- (iii) to finite populations.

(Back)

# Convergence to an Equilibrium

## Proposition

If  $\Gamma$  is  $2 \times 2$  game with unique Nash eqm,

- (i)  $\{\sigma^n\}_n$  converges to a sequential sampling eqm; and
- (ii) this sequential sampling eqm is globally asymptotically stable.

(Back)

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### Data from Friedman & Ward (2019)

Subjects play matching pennies with varying  $\delta_M$ ; randomly matched but roles fixed

1st stage: actions elicited; each game played  $\times 2$

2nd stage: action *or* actions + (mean) beliefs elicited; each game played  $\times 5$

Decision time recorded

Opponents in 2nd stage are drawn from 1st stage

Incentive compatible elicitation: variant BDM (Karni '09)

Payoffs = prob. points towards prizes of \$10

No feedback; game order randomised

(Back)

## Data from Friedman & Ward (WP)

### Caveats

Some evidence suggesting learning between 1st and 2nd stages

Decision time recorded, subjects forced to wait minimum 10 sec before answering

Elicitation of actions and beliefs is sequential instead of simultaneous

Subjects do not always best-respond to reported beliefs: best-response rate 75%

[\(Back\)](#)

## Selection of Nash Equilibria

**Weak selection:** Which Nash equilibria can we select *with some* priors?

Example: Nash equilibria with weakly dominated actions not reachable

(Back)

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$\sigma$  PSNE can be selected for some  $(\mu_i)_i$  **if and only if**

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(Back)

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**Weak selection:** Which Nash equilibria can we select *with some* priors?

Example: Nash equilibria with weakly dominated actions not reachable

$\sigma$  PSNE can be selected for some  $(\mu_i)_i$  **if and only if**

$\sigma$  does not involve weakly dominated actions.

**Strong selection:** Which Nash equilibria can we select *with any* priors

All that sat. robustness condition:

action remains best-response to any small perturbation of others' eqm distrib.

Weaker than strict Nash eqa, stronger than trembling-hand perfect.

(Back)