Speed, Accuracy, and Complexity

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Economics Micro Theory Seminar

Carnegie Mellon University – University of Pittsburgh

16 April 2026

Mistakes and Complexity

Why do we care?

Dominated choices: lottery choice, dominant strategy mechanisms,

health insurance plans, pension plans, mortgages, etc.

Cognition and Complexity

A leading explanation for behavioural 'biases':

Cognitive limitations/costs and problem complexity.

+ Complex problems $\rightarrow$ + Strain on cognitive resources $\rightarrow$ + Mistakes.

More complex problem if more mistakes and/despite greater cognitive effort.

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– Ability $\rightarrow$ + Cognitive costs $\rightarrow$ + Mistakes.

Less able agents treat same problem as if more complex.

Mistakes and Complexity

Inferring Complexity

Understand when decision problem challenging and make it simpler (or not).

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Usual proxy: **response times**

(Stroop 35; Hick 52; Shepard Metzler 71; Treisman Gelade 80; Bassili Scott 96; Roitman Shadlen 02; Wilson et al. 10; Murawski Bossaerts 16; Franco et al. 21; Gill Prowse 23; Hong Stauffer 23; ...)

Easy choices will produce fast and accurate responses, while difficult ones will be time consuming and poorly efficient

(Cerreia-Vioglio Maccheroni Marinacci Rustichini 22)

Faster and better is easier; slower and worse is more complex.

From Slower is Worse to Slower is More Complex

Speed-Accuracy Trade-off.

More time, more information: better choices, but more costly

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Slower is worse.

Less decisive info $\rightarrow$ Slower and worse choices.

Close to indifferent $\rightarrow$ Hard choice.

Seq sampling: conflicting info, closer to indifferent, higher value info, sample +
(Fudenberg Strack Strzalecki 18; Gonçalves WP).

Evidence: food choice (Krajbich Lu Camerer Rangel 12; Clithero 18; Alós-Ferrer Fehr Netzer 22); lotteries (Alós-Ferrer Garagnani 20); global games (Schotter Trevino 21); matching pennies (Gonçalves WP); bargaining (Konovalov Krajbich 20).

From Slower is Worse to Slower is More Complex

Speed-Accuracy Trade-off.

More time, more information: better choices, but more costly.

Slower is worse.

Less decisive info $\rightarrow$ Slower and worse choices.

Slower is more complex.

Slower and worse implies more complex.

Slower implies worse.

Ergo, slower reflects more complex.

Syllogism implicit or explicit in use of response times as proxy.

Rethinking the Question

Runs against intuition: **Slower cannot always be worse and more**.

If too complicated, not going to try too hard.

The importance of being aware: **Need to recognise how complex problem is.**

Speed, Accuracy, and Complexity

This paper: reconcile intuitive ambiguity on how problem complexity relates to speed and accuracy.

Revisit canonical sequential sampling model (Wald 45; Dvoretzky Kiefer Wolfowitz 53) and operationalise *problem complexity*.

Four Main Results:

(1) With exogenous stopping, time increasing in complexity.

(2) With endogenous optimal stopping, time *non*-monotone in complexity.

(3) Extend model to speak to heterogeneous ability.
Higher ability: faster in simple problems, slower in complex problems.

(4) Provide method to infer complexity and ability from choices.
Subsidies more effective in more complex problems and for less able DMs.

Overview

1. Setup
2. Speed and Accuracy under Exogenous Stopping
   - Comparative Statics in Problem Complexity under Exogenous Stopping
   - Speed-Accuracy Complementarity in Related Models
3. Speed and Accuracy under Optimal Stopping
   - Characterising Optimal Stopping
   - Comparative Statics in Problem Complexity under Optimal Stopping
   - Speed-Accuracy Non-Monotonicity in Related Models
   - Reconciling Empirical Evidence
4. Speed, Accuracy, Complexity... and Ability
   - Effort, Ability, and Cost Complexity
   - Comparative Statics in Ability
5. Identifying Complexity (and Ability)
6. Final Remarks

Setup

Setup

Binary choice problem.

DM faces binary decision problem $\alpha \in \{a, b\}$

Unknown state determines which alternative is best $\theta \in \{a, b\}$; $p_0 := \mathbb{P}(\theta = a) = 1/2$.

Payoffs $u(\alpha, \theta)$ Matching better than not $u(\theta, \theta) > u(\theta', \theta)$

Expected payoffs $u(\alpha, p) := \mathbb{E}_{\theta \sim p}[u(\alpha, \theta)]$.

Setup

Sequential sampling: Prior to choosing, DM can acquire info about θ at flow cost $c > 0$.

$X_t = s_\theta \mu t + \sigma B_t$; B_t std Brownian motion; $s_\theta = 1$ if $\theta = a$, and $s_\theta = -1$ if $\theta = b$; $\mu, \sigma > 0$ drift rate and volatility.

Get signal $z_{dt} \sim N(dt\, s_\theta\, \mu\; dt\, \sigma^2)$ every Δ. $X_t \approx$ sum of z_{dt}.

$p_t := \mathbb{P}(\theta = a \mid \mathcal{F}_t^X)$. $\text{logit}(p_t) = 2 s_\theta (\sigma/\mu)^{-2}\, t + 2(\sigma/\mu)^{-1} B_t$.

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Stopping Problem: DM chooses stopping time τ adapted to $\mathcal{F}_t^X$.

Upon stopping, maximises expected utility: $\alpha_\tau \in \arg\max_{\alpha \in A} u(\alpha, p_\tau)$.

Expected payoffs: $\mathbb{E}[\max_\alpha \mathbb{E}[u(\alpha, \theta) | \mathcal{F}_\tau^X] - c\tau]$.

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Common Interpretation: Stylised representation of deliberation or reasoning.

Benchmark in modelling neural basis of decision-making (Ratcliff 78; Fehr Rangel 11; Ratcliff et al. 16; Shadlen Shohamy 16; Gold Shushruth, Zylberberg Shadlen 22).

Applied to wide range of situations: consumer choice (Krajbich et al. 11, 12; Branco Sun Villas-Boas 12; Clithero 18), group deliberation and social learning (Reshidi et al. 25; Frydman Krajbich 22), brand recognition and advertising (Alós-Ferrer 18; Chiong et al. 23), strategic choice (Schotter Trevino 21), persuasion (Orlov et al. 20; Escudé Sinander 23), policy experimentation (Callander 14; McClellan 24; Wong 25).

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Problem Complexity: noise-to-signal ratio σ/μ.

How difficult it is to learn which alternative is better from the available evidence.

- Savings products vs structured financial products.
- Compare items with and without attribute-by-attribute comparison tools.

Related Notions of Complexity (cf. Oprea 20):

Computational complexity: min resources required to solve a problem.

Sample complexity: sample size required to attain given accuracy level.

Cost complexity: costs of *specific DM* associated with specific choice procedures when faced with specific problem.

Assume for now: **DM knows ex ante problem complexity.**

Speed and Accuracy under Exogenous Stopping

Exogenous Stopping

Drift-Diffusion Model

Ratcliff 78: highly influential paper in cognitive sciences, thousands of cites.

Model time and choices via BM with drift under exogenous stopping decision.

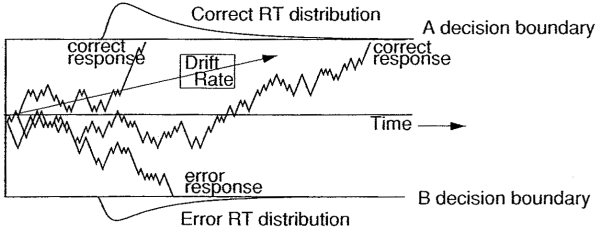

Figure adapted from Ratcliff McKoon 08.

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Exogenous Stopping Benchmark.

Stopping thresholds: $(\underline{p}_t, \overline{p}_t)$. Stopping time: $\tau := \inf\{t \geq 0 \mid p_t \notin (\underline{p}_t, \overline{p}_t)\}$.
Choice: $\alpha_\tau = a$ if belief hits upper threshold $p_\tau = \overline{p}_\tau$; ow $\alpha_\tau = b$ if $p_\tau = \underline{p}_\tau$.

E.g., stopping upon reaching pre-determined time-dependent level of certainty, or securing satisficing level of expected payoff.

Assumptions: $\tau < \infty$ a.s. (e.g., thresholds grow sublinearly in log-odds);
Prior contained in continuation region, $p_0 \in (\underline{p}_t, \overline{p}_t)$ whenever nonempty:

$$\forall T > 0 \text{ such that } (\underline{p}_T, \overline{p}_T) \neq \emptyset,\ \exists \varepsilon_T > 0 :\ \forall t \leq T, (p_0 - \varepsilon_T, p_0 + \varepsilon_T) \subset (\underline{p}_t, \overline{p}_t).$$

Comparative Statics in Problem Complexity under Exogenous Stopping

Theorem 1

(i) Stopping time τ (FOSD) increases in problem complexity.

(ii) If continuation region $(\underline{p}_t, \overline{p}_t)$ shrinks (expands) with t, then accuracy is decreasing (resp., increasing) in problem complexity.

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Proof Intuition: part (i)

Key observation: More complex problem $\equiv$ run same belief process on slower clock.

Two channels of greater complexity: (a) belief process evolves more slowly;
(b) effectively compare same realised path at t with thresholds at $t' > t$.

Forces go together when thresholds expand over time, not when they shrink over time.

Exogenous Stopping

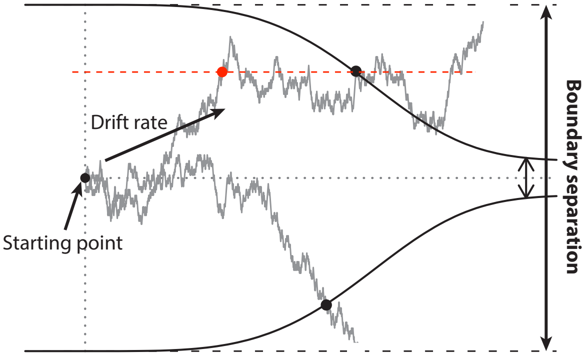

Figure adapted from Forstmann Ratcliff Wagenmakers 16.

More complex $\rightarrow$ slower clock and compare value at t with later thresholds.

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Key observation: More complex problem $\equiv$ run same belief process on slower clock.

Main steps in proof:

(1) PDE characterisation of survival prob. as backward parabolic value function.
(2) Monotonicity wrt time under mollified (smooth) boundary conditions.
(3) Couple belief processes with higher/lower complexity using mollified PDE.
(4) FOSD shift on restricted domain via optional stopping argument and pass to limits.

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Proof Intuition: part (ii)

With symmetric boundaries ($\overline{p}_t = 1 - \underline{p}_t$), accuracy = posterior, i.e., $\mathbb{P}(\alpha_\tau = \theta) = \overline{p}_\tau$.

$\overline{p}_t$ is decreasing (increasing) in t + FOSD shift $\Longrightarrow$ lower (higher) accuracy.

Extend beyond symmetric boundaries.

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Extend beyond symmetric boundaries.

Main steps in proof: $\sigma/\mu = \kappa_H > \kappa_L$

(1) Time shift in belief process: $p^\kappa_t = p_{(\kappa)^{-2}t}$.
(2) Shrinking thresholds + coupling argument: $(\kappa_H/\kappa_L)^2 \tau^{\kappa_H} \geq \tau^{\kappa_L}$.
(3) Accuracy: $|p^\kappa_{\tau^\kappa} - 1/2| + 1/2$ submartingale.
(4) Optional stopping: $\mathbb{E}[|p^\kappa_{\tau^\kappa} - 1/2|]$ decreases in κ.

Speed-Accuracy Complementarity in Related Models

Uncertain Complexity (Fudenberg Strack Strzalecki 18).

DM's uncertain about problem complexity. Even if optimal given prior, stopping thresholds as if exogenous relative to *realised* complexity.

Their paper: accuracy *conditional on stopping time* decreasing.

This paper: new comparative statics; *unconditional* accuracy decreases and stopping time increases in realised complexity.

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Robustness and Misspecification (Wald 47).

Maxmin DM doesn't know $\sigma/\mu \in [\underline{\kappa}, \overline{\kappa}]$.
Worst-case scenario, $\overline{\kappa}$; stopping rule invariant wrt realised σ/μ.
Similarly: DM behaves optimally but misspecified about σ/μ.

Accuracy decreases and stopping time increases in realised complexity.

Speed and Accuracy under Optimal Stopping

Optimal Stopping

From Exogenous to Optimal Stopping.

Optimal Stopping: expect DM to trade-off accuracy and effort/time.

Behaviour depends on incentives and problem complexity σ/μ.

$$V(p_0) := \sup_{\tau} \mathbb{E}[\max_{\alpha} \mathbb{E}[u(\alpha, \theta)|\mathcal{F}_{\tau}^{X}] - c\tau].$$

Characterising Optimal Stopping

Proposition

Optimal stopping time is $\tau^* = \inf\{t \geq 0 : p_t \notin (\underline{p}, \overline{p})\}$, where $\underline{p}$ and $\overline{p}$ uniquely solve:

$$(\sigma/\mu) = m_1(\overline{p}, \underline{p}) \qquad \text{and} \qquad \tilde{p} = m_2(\overline{p}, \underline{p}).$$

Solution to optimal stopping problem known (Peskir Shiryaev 06, Thm 21.1).

Quickly work out intuition and define key variables.

Characterising Optimal Stopping

Proposition

Optimal stopping time is $\tau^* = \inf\{t \geq \mathbf{0} : p_t \notin (\underline{p}, \overline{p})\}$, with constant thresholds $\underline{p}$ and $\overline{p}$ uniquely determined.

Step 1: Separate relative from absolute incentives.

Indifference $\tilde{p} : u(a, \tilde{p}) = u(b, \tilde{p})$; captures relative incentives.

Stakes $\delta := (u(a,a) - u(b,a)) + (u(b,b) - u(a,b))$; captures absolute incentives.

Auxiliary problems: $u_a(\alpha, p) := \delta \mathbf{1}\{\alpha = a\}(p - \tilde{p})$ and $u_b(\alpha, p) := \delta \mathbf{1}\{\alpha = b\}(\tilde{p} - p)$.

Affine transformations: $u = u_a + u(b, \cdot) = u_b + u(a, \cdot)$.

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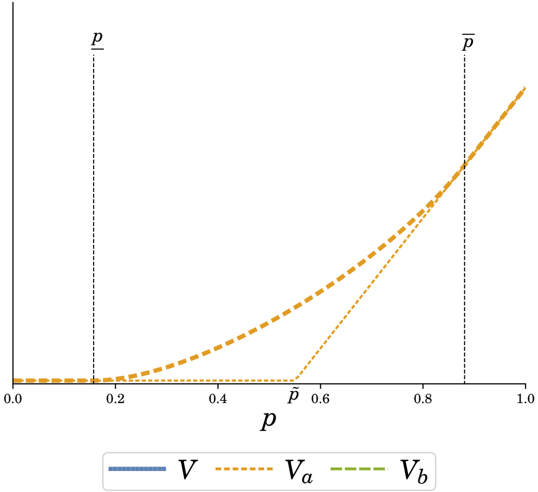
Optimal stopping time is $\tau^* = \inf\{t \geq 0 : p_t \notin (\underline{p}, \overline{p})\}$, with constant thresholds $\underline{p}$ and $\overline{p}$ uniquely determined.

Step 1: Separate relative from absolute incentives.

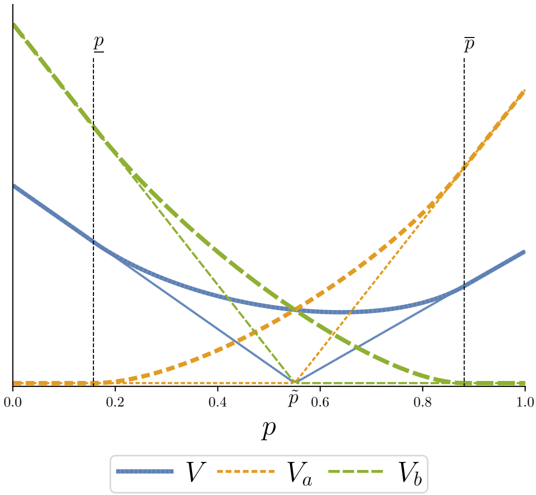
Step 2: Constant thresholds $(\underline{p}, \overline{p})$.

Characterising Optimal Stopping



V_a increasing, $V_a \geq \max_\alpha u_a \geq \mathbf{0}$, and $V_a = \mathbf{0} \iff$ stop and choose b.
$\exists!$ threshold $\underline{p}$ below which immediately stopping and choosing b is optimal.
Symmetric argument for $\overline{p}$.

Characterising Optimal Stopping



Optimal stopping time is the same in original and auxiliary problems.
Affine transformations: $V = V_a + u(b, \cdot) = V_b + u(a, \cdot)$.

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Step 1: Separate relative from absolute incentives.

Step 2: Constant thresholds $(\underline{p}, \overline{p})$.

Step 3: Thresholds from differential characterisation of value function.

Value function V is unique viscosity solution to free-boundary problem:

$$\underbrace{c}_{\text{Mg Cost Continuing}} = \underbrace{2(\sigma/\mu)^{-2}\,(p(1-p))^2\, V''(p)}_{\text{Mg Benefit Continuing}} \quad \text{on } p \in (\underline{p}, \overline{p}) \tag{HJB}$$

$$\underbrace{V(p)}_{\text{Value Stopping}} = \underbrace{v(p)}_{\text{Max Payoffs}} \quad \text{on } p \notin (\underline{p}, \overline{p}) \tag{BC}$$

Recover thresholds from value function; $\underline{p}$ and $\overline{p}$ uniquely solve:

$$(\sigma/\mu) = m_1(\overline{p}, \underline{p}) \qquad \text{and} \qquad \tilde{p} = m_2(\overline{p}, \underline{p}).$$

Comparative Statics in Problem Complexity under Optimal Stopping

Higher complexity/worse information $\uparrow \sigma/\mu$

$\implies$ need more time to get to same level of accuracy

$\implies$ lower marginal benefit of engaging for longer

Theorem 2

(i) Expected optimal stopping time $\mathbb{E}[\tau^*]$ is non-monotone and quasi-concave in problem complexity.

(ii) Accuracy is decreasing in problem complexity.

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Result doesn't depend on u

e.g. whether it is more important to match on particular state, or that stakes are higher/lower.

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Proof Intuition

Key observation: greater complexity induces narrower stopping thresholds.

Accuracy: DM acquires less info in Blackwell sense.

Speed: two opposing forces.

(a) beliefs take longer to exit a fixed continuation region, but (b) continuation region itself becomes narrower.

Force (a) dominates when simpler problems become more complex;
Force (b) dominates when problem complexity high from the start.

Proof follows from straightforward optional stopping arguments together with applications of implicit function theorem.

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Since $\mathbb{E}[\text{logit}(p_\tau) \mid \theta] = \mathbb{P}(\alpha_\tau = a \mid \theta)\ \text{logit}(\overline{p}) + \mathbb{P}(\alpha_\tau = b \mid \theta)\ \text{logit}(\underline{p})$,

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$$\Longrightarrow \mathbb{E}[\tau] = \frac{1}{4m_1(\overline{p}, \underline{p})} \frac{(1 + \overline{p}/(1-\overline{p}))(1 - \underline{p}/(1-\underline{p}))}{2(\overline{p}/(1-\overline{p}) - \underline{p}/(1-\underline{p}))} (\text{logit}(\overline{p}) - \text{logit}(\underline{p}))$$

$$=: T(\overline{p}, \underline{p})$$

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Main steps in proof:

(3) Non-monotonicity of RT: immediate:

$$\sigma/\mu \to 0 \implies (\underline{p}, \overline{p}) \to (0,1) \implies T(\overline{p}, \underline{p}) \to 0$$

$$\sigma/\mu \to \infty \implies (\underline{p}, \overline{p}) \to (\tilde{p}, \tilde{p}) \implies T(\overline{p}, \underline{p}) \to 0.$$

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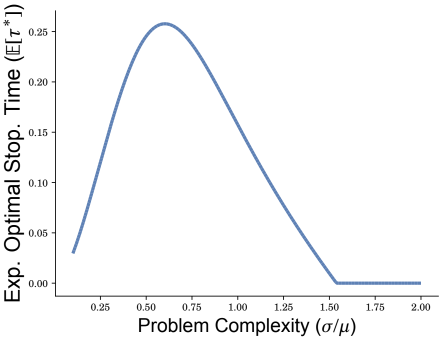
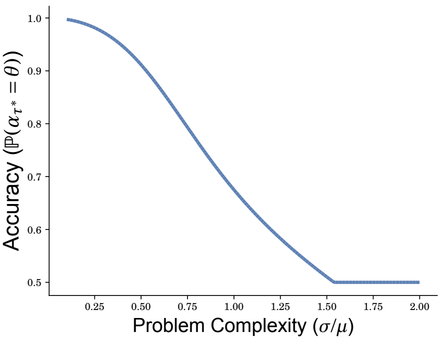
(4) Single-peaked: Implicit function (using m_2);

$T(\overline{p}, \underline{p}(\overline{p}))$ quasiconcave in $\overline{p}$ + monotonicity $\overline{p}$ in μ/σ
$\implies \mathbb{E}[\tau^*]$ single-peaked in μ/σ.

Speed-Accuracy Non-Monotonicity





Speed-Accuracy Non-Monotonicity in Related Models

Preference Intensity and Discounting.

Speed of learning proportional to preference intensity $(\sigma/\mu)\,\delta^{-1}$ (Fehr Rangel 11).

Discounting instead of flow cost: same results.

Costly Information Acquisition (Details)

(Matejka McKay 15; Steiner Stewart Matejka 17; Caplin Dean 15; Caplin Dean Leahy 22).

DM can choose arbitrary info structures. Maintain two operational assumptions:

- problem complexity acts as a time-scaling parameter, and
- cost of info is monotone in amount of time-equivalent information acquired.

Non-monotone speed-complexity relation emerges; inverse-*U* shape needs more.

Speed-Accuracy Non-Monotonicity in Related Models

Uncertain versus Recognised Complexity (Fudenberg Strack Strzalecki 18).

DM's uncertain about problem complexity. Even if optimal given prior, stopping thresholds as if exogenous relative to *realised* complexity.

Accuracy decreases and stopping time increases in realised complexity.

However, analogue of comparative statics is wrt prior, not realised complexity.

$\sigma/\mu + \rho\varepsilon$, $\varepsilon \sim f$.

For low enough uncertainty ρ, still obtain non-monotone speed-complexity relationship.

Reconciling Empirical Evidence

Speed-Complexity Monotonicity.

DM cannot adjust to problem complexity independently of learning the optimal solution: as if exogenous stopping.

Computational/numerical problems: identify larger numerical value of numbers/sums (Moyer Landauer 67; Buckley Gillman 74; Krajcsi et al. 16).

Perception problems: identify dominant colour/movement in set of items (Linde Paivio 79; Maanen et al. 11).

Smaller difference between alternatives makes it more complex; but indistinguishable from learning optimal solution.

More complex entails slower and worse.

Reconciling Empirical Evidence

Speed-Complexity Monotonicity.

DM cannot adjust to problem complexity independently of learning the optimal solution: as if exogenous stopping.

Speed-Complexity Non-Monotonicity.

Problem complexity separable from determinants of optimal action.

DM can adjust to problem complexity independently of learning the optimal solution: optimal stopping.

E.g., number summands or size of set considered (Gonçalves Nunnari Zarate-Pina WP).

Also: nature of T/F questions (Wright Ayton 88) and features of lottery choice problems and contingent reasoning problems (Agranov Schotter Trevino WP).

Speed, Accuracy, Complexity... and Ability

Effort and Ability

Speed and Ability:

Slower as more sophisticated, documented relationship:

financial choices (Darriet et al. 20); dominance-solvable games (Rubinstein 07, 16; Agranov Caplin Tergiman 15; Alós-Ferrer Buckenmaier 21; Esteban-Casanelles Gonçalves WP; Gill Prowse 23); public goods (Recalde Riedl Vesterlund 18)

Intuition: + ability $\rightarrow$ + effort $\rightarrow$ + time

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Intuition: + ability $\rightarrow$ + effort $\rightarrow$ + time

Faster as more sophisticated: huge literature in psychology dating back to Thorndike, Bregman, Cobb, and Woodyard 1926

other things being equal, if intellect A can do at each level the same number of tasks as intellect B, but in a less time, intellect A is better.

Optimal Level of Effort

Cost Complexity: Costs idiosyncratic; time not necessarily good measure of effort spent in task.

- Cost of time not necessarily $\propto$ cost of effort.
- Depends on DM ability and their approach to problem.
  - Spend a lot of time and exert little effort or vice-versa.

Effort Control:

- Effort scales signal: $dX_t = \sqrt{e_t}\, s_\theta\, \mu \; dt + \sigma\, dB_t$.
- Time cost c depends on effort level e_t at t: $\quad c, c', c'' > 0$.

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Ability:

Greater ability: lower time-cost per effort. $c(e_t/\lambda)$, $\lambda > 0$.

Ability does not affect time and accuracy directly; only through effort choice.

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DM's Problem: choose τ, $(e_t)_t$ to maximize

$$\mathbb{E}\left[\max_{\alpha} \mathbb{E}[u(\alpha, \theta) \mid \mathcal{F}_t^X] - \int_0^{\tau} c(e_t/\lambda)\, dt\right].$$

Comparative Statics in Ability

Theorem 3

(i) Expected optimal stopping time $\mathbb{E}[\tau^*]$ is non-monotone and quasi-concave in ability.
(ii) Accuracy is increasing in DM's ability.
(iii) Furthermore, expected optimal stopping time has single-crossing property in ability and problem complexity.

Proof Intuition

Setup similar to Moscarini Smith (01).

Optimal stopping: $\underbrace{e^*_\lambda 2(\sigma/\mu)^{-2}(p(1-p))^2 V''(p)}_{\text{Mg Benefit Continuing}} = \underbrace{c(e^*_\lambda/\lambda)}_{\text{Mg Cost Continuing}}$ (from HJB).

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$e^* c'(e^*) = c(e^*)$ has unique solution.

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$\Longrightarrow$ constant effort $e_\lambda^* = \lambda e^*$; drift $\sqrt{\lambda e^*}\mu$; flow cost $c(e_\lambda^*/\lambda) = c(e^*)$.

Maps back to previous problem with idiosyncratic complexity $\kappa_\lambda = (\sigma/\mu)(\lambda e^*)^{-1/2}$.

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Higher ability $\equiv$ **Simpler problem**.

Single-crossing:

in simple problems, DM with higher ability chooses faster and better;

in complex problems, DM with higher ability chooses slower and better.

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Discounting (Moscarini Smith 01).

Discounting on top of flow cost. Effort no longer constant over time.

Still: Higher ability, wider thresholds, higher accuracy.

Higher ability, higher effort, non-monotonicity in problem complexity and ability.

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Reconciling Empirical Evidence:

Inverted-*U* shaped speed-IQ (Lindley et al. 95).

Faster responses more indicative of higher ability in easier tasks (Dodonova Dodonov 13; Goldhammer et al. 15).

Identifying Complexity (and Ability)

Inferring Complexity (and Ability)

Back to inferring complexity and ability...

Complexity: cognition/info processing underlying several 'biases' in behaviour:

(Luce, Wilcox, Rubinstein, Spiegler, Frydman, Jin, Bossaerts, Oprea, Enke, Zimmermann, Graeber, Salant, Agranov, Esponda, Vespa, Yuksel, Martínez-Marquina, Niederle, Puri, Alaoui Penta, ...; also Li, Borgers Li, Pycia Troyan, Camara, ...)

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There is a correct answer (and analyst knows it):

Choices tend to identify complexity (even in heterog. populations).

(Avg) Accuracy decreasing in complexity.

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There is no correct answer/Analyst doesn’t know it:

Response time often used as proxy, but not always appropriate.

Non-monotone relationship creates inference problem.

Going to higher moments doesn’t help.

Heterogeneous population: not even quasiconcave.

What to do?

Relative Incentives, Complexity, and Ability

Subsidies and Relative Incentives: subsidise *b* by 1 penny.

Affects relative incentives ($\tilde{p}$), not absolute incentives (δ).

Makes DM more likely to choose *b* and do so faster
and less likely to choose *a* and do so slower (Gonçalves WP).

Pushes both stopping thresholds up: $\uparrow \tilde{p} \implies \uparrow \overline{p}, \underline{p}$.

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Subsidies more effective if have greater effect on choices $\frac{\partial}{\partial \tilde{p}} \mathbb{P}(\alpha_\tau = b)$, when $\mathbb{P}(\alpha_\tau = b) \in (0, 1)$.

Identifying Complexity and Ability

Theorem 4

Subsidies are more effective in more complex problems and less able DMs.

Intuition: 1 penny more for *b* pushes stopping thresholds up.

If problem simple enough, then close to sure, very wide continuation region; won't affect choice prob. a lot.

If problem very hard, narrow continuation region; affects choice prob. much more.

Proof: optional stopping + implicit function theorem.

Identifying Complexity and Ability

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If problem very hard, narrow continuation region; affects choice prob. much more.

Proof: optional stopping + implicit function theorem.

Limitations: hold fixed (distribution of) absolute incentive levels / stakes fixed.

Holds beyond DDM framework — e.g., Shannon costs (Ambuehl Ockenfels Stewart 25) — but not in general UPS costly info acquisition.

Final Remarks

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This paper: reconcile intuitive ambiguity on how problem complexity relates to speed and accuracy.

Revisit canonical sequential sampling model (Wald 45; Dvoretzky Kiefer Wolfowitz 53) and operationalise *problem complexity*.

Four Main Results:

(1) With exogenous stopping, time increasing in complexity.

(2) With endogenous optimal stopping, time *non*-monotone in complexity.

(3) Extend model to speak to heterogeneous ability.
Higher ability: faster in simple problems, slower in complex problems.

(4) Provide method to infer complexity and ability from choices.
Subsidies more effective in more complex problems and for less able DMs.

Follow-on work: test comparative statics (Gonçalves Nunnari Zarate-Pina WP).

Diverse problem domains: perception, computation, logic, prediction, inference.
Monotonicity and non-monotonicity of time in un/recognised complexity.
Provide clear evidence for comparative statics.

Speed, Accuracy, and Complexity

Duarte Gonçalves

University College London

Economics Micro Theory Seminar

Carnegie Mellon University – University of Pittsburgh

16 April 2026

Overview

1. Setup

2. Speed and Accuracy under Exogenous Stopping
   - Comparative Statics in Problem Complexity under Exogenous Stopping
   - Speed-Accuracy Complementarity in Related Models

3. Speed and Accuracy under Optimal Stopping
   - Characterising Optimal Stopping
   - Comparative Statics in Problem Complexity under Optimal Stopping
   - Speed-Accuracy Non-Monotonicity in Related Models
   - Reconciling Empirical Evidence

4. Speed, Accuracy, Complexity... and Ability
   - Effort, Ability, and Cost Complexity
   - Comparative Statics in Ability

5. Identifying Complexity (and Ability)

6. Final Remarks

General Costly Information Acquisition

(Back)

From Dynamic to Static

Stopping time $\tau \mapsto$ Distribution over posterior beliefs $p_\tau \sim \pi$.

Benefit $B(\pi) \equiv \mathbb{E}_{p_\tau \sim \pi} v(p_\tau)$ and Cost $\kappa C(\pi) \equiv c\mathbb{E}[\tau]$, with $\kappa = \left(\frac{\sigma}{\mu}\right)^2$.

Static costly information acquisition as ex-ante expected cost of stopping time (Morris Strack WP).

General Costly Information Acquisition

(Back)

Info structures π distrib over posterior beliefs; $\Pi := \{\pi \in \Delta(\Delta(\Theta)),\ \mathbb{E}_\pi[p] = p_0\}$.

$\pi_U \in \Pi$: fully uninformative; $\pi_I \in \Pi$: fully informative.

$\geq_B$: Blackwell order.

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Costly Info Acquisition

Cost of info $C : \Pi \to \overline{\mathbb{R}}$; increasing in $\geq_B$, continuous, $C(\pi_U) \equiv 0$, strictly convex.

Benefit of info $B : \Pi \to \mathbb{R}$; bounded, continuous, increasing in $\geq_B$, linear.

Value of info $v(\pi; \kappa) := B(\pi) - \kappa C(\pi)$.

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Value of info $v(\pi; \kappa) := B(\pi) - \kappa C(\pi)$.

Optimal info acquisition $V(\kappa) := \max_\pi v(\pi; \kappa)$.

$$\pi^*(\kappa) := \arg\max_\pi v(\pi; \kappa), \quad C^*(\kappa) := C(\pi^*(\kappa)).$$

General Costly Information Acquisition

(Back)

Assumption: Time increasing in total cost of info: $T^* = g(\kappa C^*(\kappa)), \ \kappa \geq 0$.

κ measure of problem complexity; scales (mg) cost info.

Wald $\kappa = (\sigma/\mu)^2$, even with many states (Morris & Strack WP) (g = id).

General CIA problems with cost scaling.

Non-Monotonicity with Costly Information Acquisition

(Back)

Proposition

T^* is non-monotone in κ.

Non-Monotonicity with Costly Information Acquisition

(Back)

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T^* is non-monotone in κ.

Proof Sketch

$$\kappa_n \searrow 0 \implies V(\kappa_n) \nearrow B(\pi_I) \text{ (continuity from Berge)} \implies \kappa_n C(\pi^*(\kappa_n)) \searrow 0.$$

$$\kappa_n \nearrow \infty \implies V(\kappa_n) \searrow B(\pi_U) \implies \kappa_n C(\pi^*(\kappa_n)) \searrow 0.$$

$$T^* = g(\kappa C(\pi^*(\kappa))) \to g(0) \quad \text{as} \quad \kappa \searrow 0.$$

Note: lower cost ($\downarrow \kappa$) does not imply (even weakly) + informative π^*.

Non-Monotonicity with Costly Information Acquisition

(Back)

Proposition

T^* is non-monotone in κ.

Non-monotonicity of time general property.

Conditions for single-peaked T^*.

Quasiconcavity in Binary Problems

(Back)

Back to Binary problems: $\Theta = A = \{0, 1\}$, $p_0 = \tilde{p} = 1/2$.

$C(\pi) := \mathbb{E}_{\pi}[c(p)] - c(p_0) \quad c \in \mathcal{C}^2$, strictly convex.

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$C(\pi) := \mathbb{E}_\pi[c(p)] - c(p_0)$ $\quad c \in \mathcal{C}^2$, strictly convex.

At most two posteriors, $\overline{p} = 1 - \underline{p}$, equal prob.

FOC: $c'(\overline{p}) - c'(1 - \overline{p}) = 1/\kappa$.

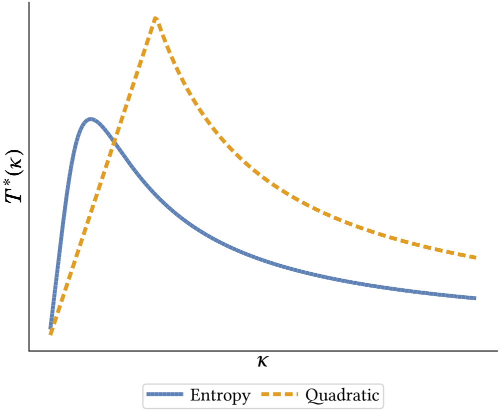
Quasiconcavity of $T^* \iff$ Quasiconcavity of $c(p)/(c'(p) - c'(1 - p))$ on $[1/2, 1]$.

Not ensured in general. (Constructing counterexamples).

Holds for Wald, entropy, log costs.

Quasiconcavity in Binary Problems

(Back)



Entropy costs: $c(p) := p\log(p) + (1-p)\log(1-p)$.

Quadratic costs: $c(p) := (p - p_0)^2$.

Quasiconcavity in Binary Problems

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Quasiconcavity of T^* $\iff$ Quasiconcavity of $c(p)/(c'(p) - c'(1-p))$ on $[1/2, 1]$.

Constructing Counter-examples:

$\psi : [1/2, 1] \to \mathbb{R}_+ \cup \{\infty\}, \quad \psi \in \mathcal{C}^1(1/2, 1]$,

(i) $\psi \geq 0$, (ii) $\psi' < 1$, (iii) $\lim_{x \downarrow 1/2} \psi(x) = \infty$, (iv) $\exists \hat{p} \in (1/2, 1)$: $\psi'(\hat{p}) = 0 < \psi''(\hat{p})$.

Symmetrise ψ about 1/2.

Define $c(p) := c(1/2) \exp\left(\int_{1/2}^{p} (\psi(x))^{-1} \, dx \right)$.

$\implies$ $c > 0$ and strictly convex, symmetric about 1/2.

$c(p)/(c'(p) - c'(1-p)) = c(p)/2c'(p) = \psi(p)/2$ **not** quasiconcave .

$\implies$ T^* **not** quasiconcave (still non-monotone).

E.g., $\psi(p) := (p - 1/2)^{-1} + \gamma p, \quad p \in (1/2, 1]$ and $\gamma \in (4, 5)$.

Quasiconcavity in General

(Back)

General condition: $-\kappa V''(\kappa)/V'(\kappa)$ crosses 1 exactly once.

Intuition

(1) **Convexity:** V convex; V', V'' exist a.e. (Alexandrov).

(2) **Total cost:** $\kappa C(\pi^*(\kappa)) = -\kappa V'(\kappa)$ (envelope).

(3) **Single-peakedness:** $-\kappa V''(\kappa)/V'(\kappa)$ crosses 1 at most (exactly) once.