ECON0106: **Microeconomics**

5. Expected Utility*

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1. Overview

So far we have looked at situations in which the outcomes of choices are deterministic, that is, where the decision-maker knows exactly all the consequences associated to all alternatives and makes the utility-maximizing choice.

Here is an example of what is missing in our model: Say a decision-maker sets out to buy a computer. It just so happens that new computers may or may not be faulty. The decision-maker would prefer the computer not to be faulty, so our model treats faulty computer and not faulty computer as two different elements. The issue is that, ex-ante, one may know how likely it is that a computer is faulty, but not know whether a computer is or is not faulty. How should we take that into account?

In this lecture we will address exactly this question: how to have a useful utility representation of preferences over probability distributions. We will work with the assumption that the agent is facing a situation in which these probabilities are *known* and *objective*. In the case above, this amounts to knowing the exactly probability that the computer is faulty. When probabilities over outcomes are known, we say the agent is facing **risk**. This is an important assumption: it allows a researcher to treat these probabilities as something observable.

There are of course cases in which probabilities over outcomes are unknown. For instance, people may disagree about the probability with which a given team wins a sports match. When probabilities are not known, we say the agent is facing **uncertainty**[1] and their beliefs about the likelihood of the outcome are *subjective*. This is a crucial difference, as we cannot directly observe a person's beliefs. We defer a treatment of uncertainty to later on and focus first on choice under risk.

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[1] The term "ambiguity" is also used in the literature.

2. Setup

Let X finite **outcome space**. An element $x \in X$ entails a complete description of all relevant aspects of the environment. That is, in the example above of buying a computer, x would describe whether a particular computer is faulty or not. We denote the **set of all probability measures on** X by $\Delta(X)$, i.e., the set of all functions $p : X \to [0,1]$ such that $\sum_{x \in X} p(x) = 1$; we will occasionally call p a **lottery**. Equivalently, as X is finite — say $|X| = N$, you can think of p as a vector in a subset of $[0,1]^N$.[2] We let $\succsim$ be a preference relation on $\Delta(X)$, and we endow the set with the Euclidean metric.

A special case is that of **degenerate probability measures**, i.e., those assigning probability one to a particular outcome and we will write them differently whenever we want to emphasise that aspect. In particular, we write $\delta_x \in \Delta(X)$ to denote the probability measure that assigns probability 1 to x, where $\delta_x(x') = 1$ if $x' = x$ and zero if otherwise.

For any $\alpha \in [0,1]$, and any $p, p' \in \Delta(X)$, we write the **probability mixture** as $\alpha p + (1-\alpha)p'$ to denote the probability measure $\alpha p + (1-\alpha)p' \in \Delta(X)$ such that for any $x \in X$, $(\alpha p + (1-\alpha)p')(x) = \alpha p(x) + (1-\alpha)p'(x)$. Two things to note. First, that $\Delta(X)$ is convex with respect to mixtures. Second, that a probability mixture *is not* a probability distribution on probability distributions. That is, $\alpha p + (1-\alpha)p'$ *is not* the probability distribution that delivers p with probability α and p' with completementary probability. (in the same way that $\delta_x \neq x$). The former lives in $\Delta(X)$ (on which preferences are defined), whereas the latter lives in $\Delta(\Delta(X))$. We discuss this subtlety at the end of the notes.

3. Expected Utility

We want to have a sensible conditions that allow us to have a useful utility representation of the agent's preferences over $\Delta(X)$. From what we have already seen, we know that if, say, $\succsim$ is a continuous preference relation on $\Delta(X)$, then it has a continuous utility representation $U : \Delta(X) \to \mathbb{R}$, that is, a utility function such that $p \succsim p' \iff U(p) \geq U(p')$. This is *a* utility representation, but we want something more tractable.

Suppose we restrict X to be money amounts. One possibility is to have utility being equal to the expected value, $U(p) = \mathbb{E}_p[x]$, where $\mathbb{E}_p[x] = \sum_{x \in X} p(x)x$. Now take these two fair lotteries: p that assigns equal probability to £5 and -£5, and p' gives probability 1/2 to both £5,000 and -£5,000. Both have expected value equal to zero; however, some people can and do disagree about which is better and have strict preferences for one over the other. It would then be too restrictive to simply assume that everyone is indifferent.

[2] Or, given that it is a probability, in the $(N-1)$-dimensional simplex $\Delta^{N-1} := \{p \in [0,1]^{|X|-1} \mid \sum_{i=1}^{N-1} p_i \leq 1\}$.

Now consider p'' such that is assigns equal probability to £5,000 and -£5. A reasonable assumption would be that everyone prefers p'' to both p (and p') as the worst outcome is the same and occurs with same probability in both, and the best outcome also occurs with the same probability in both p and p'' too, but it is far better in the p'' than in p. As $x \in X$ can be anything, the expected value approach also doesn't make much sense in many cases. So we are looking for something that relaxes the expected value assumption, but retains its appeal.

In some sense, we want to capture this by having a utility representation that disentangles these two elements: objective probabilities p, and preferences over outcomes x. Expected utility does just this.

Definition 1. We say that $\succsim$ on $\Delta(X)$ has an **expected utility (EU) representation** if there is $u : X \to \mathbb{R}$ such that $\forall p, p' \in \Delta(X)$, $p \succsim p' \iff \mathbb{E}_p[u] \geq \mathbb{E}_{p'}[u]$.

The function u is called a *Bernoulli* or *von Neumann–Morgenstern utility*. As $\sum_{x \in X} p(x)u(x)$ is just an expectation with respect to p, we will use the more compact notation $\mathbb{E}_p[u]$ to denote it. Unfortunately, it is not the case that any continuous preference relation $\succsim$ on $\Delta(X)$ has a EU representation; we need something else.

3.1. Properties

Definition 2. We say that a preference relation $\succsim$ on $\Delta(X)$ satisfies **independence** if $\forall p, p' \in \Delta(X)$, $p \succsim (\succ) p'$ if and only if for any $p'' \in \Delta(X)$, and any $\alpha \in (0,1]$, $\alpha p + (1-\alpha)p'' \succsim (\succ) \alpha p' + (1-\alpha)p''$.

In essence, what independence buys us is linearity in the space of probability distributions: $p \sim p' \implies \alpha p + (1-\alpha)p' \sim p'$. This is necessary if we want to have an expected utility representation because *expectations are linear in probabilities* in this sense: $\mathbb{E}_p[u] = \mathbb{E}_{p'}[u] \implies \mathbb{E}_{\alpha p+(1-\alpha)p'}[u] = \mathbb{E}_p[u]$. On the other hand, this implies that we are ruling out strict preference for randomisation — i.e., we cannot have $p \sim p'$ and $\alpha p + (1-\alpha)p' \succ p'$.

We consider two other properties:

Definition 3. A preference relation $\succsim$ on $\Delta(X)$

(i) has the **Archimedean property** if $\forall p, p', p'' \in \Delta(X)$ such that $p \succ p' \succ p''$, there is an $\alpha, \beta \in (0,1)$ for which $\alpha p + (1-\alpha)p'' \succ p' \succ \beta p + (1-\beta)p''$;

(ii) satisfies **vNM continuity**[3] if $\forall p, p', p'' \in \Delta(X)$ such that $p \succsim p' \succsim p''$, $\exists \gamma \in [0,1]$ for which $\gamma p + (1-\gamma)p'' \sim p'$.

We can see that if $\succsim$ has an expected utility representation, then it must be vNM continuous.

[3] I am going to call it vNM continuity — where vNM stands for von-Neumann and M for Morgenstern — to distinguish it from our previous notion of preference continuity.

To see this, note that if $\mathbb{E}_p[u] \geq \mathbb{E}_{p'}[u] \geq \mathbb{E}_{p''}[u]$, then there is $\gamma \in [0,1]$ such that $\gamma\mathbb{E}_p[u] + (1-\gamma)\mathbb{E}_{p''}[u] = \mathbb{E}_{p'}[u]$. Then, by linearity of the expectation operation in p, $\gamma\mathbb{E}_p[u]+(1-\gamma)\mathbb{E}_{p''}[u] = \mathbb{E}_{\gamma p+(1-\gamma)p''}[u]$.

This next exercise will help you develop intuition on what they imply:

Exercise 1. *Let $\succsim$ be a preference relation on $\Delta(X)$.*

(i) Prove or find a counterexample:

(a) None of the three properties implies another.

(b) If $\succsim$ satisfies independence and the Archimedean property, then it is vNM continuous.

(c) If $\succsim$ satisfies independence and vNM continuity, then it has the Archimedean property.

(ii) Show that if $\succsim$ satisfies independence and the Archimedean property, then there are $\overline{p}, \underline{p} \in \Delta(X)$ such that $\overline{p} \succsim p \succsim \underline{p}$, for all $p \in \Delta(X)$.

(iii) How do these properties relate to continuity?

(a) Does continuity imply or is implied by vNM continuity or the Archimedean property?

(b) Does continuity imply or is implied by independence?

(c) Is continuity implied by independence and vNM continuity?

3.2. Expected Utility Representation Theorem

The main result for this lecture is von Neumann and Morgenstern's (1953) expected utility representation theorem:

Theorem 1. *Let X be finite and let $\succsim$ be a preference relation over $\Delta(X)$.*

(i) $\succsim$ satisfies independence and vNM continuity if and only if it admits an expected utility representation u.

(ii) If u and v are two expected utility representations of $\succsim$, then $\exists \alpha > 0$, $\beta \in \mathbb{R}$ such that $v = \alpha u + \beta$.

Proof. The "if" part of (i) was shown in the main text. For the "only if" part of (i) we break the proof into several small steps.

Step 1. As X is finite, $\exists \delta_{\overline{x}}, \delta_{\underline{x}} \in \Delta(X)$ such that $\forall \delta_x \in \Delta(X)$, $\delta_{\overline{x}} \succsim \delta_x \succsim \delta_{\underline{x}}$.

Step 2. $\forall \{p_i\}_{i\in[n]} \subseteq \Delta(X)$ and $\forall p, p' \in \Delta(X)$ such that $p \succsim p'$, we have that $\alpha_0 p + \sum_{i\in[n]} \alpha_i p_i \succsim \alpha_0 p' + \sum_{i\in[n]} \alpha_i p_i$, for any $\{\alpha_i\}_{i\in\{0\}\cup[n]} \in [0,1]^{n+1}$ such that $\sum_{i\in\{0\}\cup[n]} \alpha_i = 1$.

Proof. If $\alpha_0 \in \{0,1\}$, the claim is trivially satisfied. For $\alpha_0 \in (0,1)$, $1-\alpha_0 = \sum_{i\in[n]} \alpha_i$, define $p'' := \sum_{i\in[n]} \frac{\alpha_i}{1-\alpha_0} p_i$ (which, by convexity of $\Delta(X)$ with respect to mixtures, belongs to $\Delta(X)$).

Then, by independence,

$$\alpha_0 p + \sum_{i\in[n]} \alpha_i p_i = \alpha_0 p + (1-\alpha_0)p''$$
$$\succsim \alpha_0 p' + (1-\alpha_0)p'' = \alpha_0 p' + \sum_{i\in[n]} \alpha_i p_i.$$

□

Step 3. $\forall p \in \Delta(X)$, $\delta_{\overline{x}} \succsim p \succsim \delta_{\underline{x}}$.

Proof. Fix an order on $X = \{x_1, x_2, ..., x_n\}$, such that $x_1 = \overline{x}$ and $x_n = \underline{x}$. By Step 1 and repeated application of Step 2,

$$\begin{aligned}
\delta_{\overline{x}} = \sum_{i=1}^{n} p(x_i)\delta_{\overline{x}} &\succsim p(x_1)\delta_{x_1} + \sum_{i=2}^{n} p(x_i)\delta_{\overline{x}} \\
&\succsim p(x_1)\delta_{x_1} + p(x_2)\delta_{x_2} + \sum_{i=3}^{n} p(x_i)\delta_{\overline{x}} \succsim \cdots \\
&\succsim \sum_{i=1}^{n} p(x_i)\delta_{x_i} = p \\
&\succsim p(x_1)\delta_{\underline{x}} + \sum_{i=2}^{n} p(x_i)\delta_{x_i} \succsim \cdots \\
&\succsim \sum_{i=1}^{n} p(x_i)\delta_{\underline{x}} = \delta_{\underline{x}}
\end{aligned}$$

□

If $\delta_{\overline{x}} \sim \delta_{\underline{x}}$, set u equally constant to any number.

In the sequel, assume $\delta_{\overline{x}} \succ \delta_{\underline{x}}$.

Step 4. $\forall \alpha, \beta : 1 \geq \alpha > \beta \geq 0$, $\alpha\delta_{\overline{x}} + (1-\alpha)\delta_{\underline{x}} \succ \beta\delta_{\overline{x}} + (1-\beta)\delta_{\underline{x}}$.

Proof. By independence,

$$\left(\frac{\alpha-\beta}{1-\beta}\right)\delta_{\overline{x}} + \left[1-\left(\frac{\alpha-\beta}{1-\beta}\right)\right]\delta_{\underline{x}} \succ \left(\frac{\alpha-\beta}{1-\beta}\right)\delta_{\underline{x}} + \left[1-\left(\frac{\alpha-\beta}{1-\beta}\right)\right]\delta_{\underline{x}} = \delta_{\underline{x}}.$$

Then, again by independence,

$$\begin{aligned}
\alpha\delta_{\overline{x}} + (1-\alpha)\delta_{\underline{x}} &= \beta\delta_{\overline{x}} + (1-\beta)\left[\left(\frac{\alpha-\beta}{1-\beta}\right)\delta_{\overline{x}} + \left[1-\left(\frac{\alpha-\beta}{1-\beta}\right)\right]\delta_{\underline{x}}\right] \\
&\succ \beta\delta_{\overline{x}} + (1-\beta)\left[\left(\frac{\alpha-\beta}{1-\beta}\right)\delta_{\underline{x}} + \left[1-\left(\frac{\alpha-\beta}{1-\beta}\right)\right]\delta_{\underline{x}}\right] = \beta\delta_{\overline{x}} + (1-\beta)\delta_{\underline{x}}
\end{aligned}$$

□

Step 5. $\forall p \in \Delta(X)$, there is unique $\gamma(p) \in [0,1]$ such that $\gamma(p)\delta_{\overline{x}} + (1-\gamma(p))\delta_{\underline{x}} \sim p$.

Proof. By Step 3, $\delta_{\overline{x}} \succsim p \succsim \delta_{\underline{x}}$. vNM continuity ensures existence of a $\gamma \in [0,1]$. By Step 4, it must be unique. □

Step 6. Let $u : X \to \mathbb{R}$ be given by $u(x) = \gamma(\delta_x)$. $p \sim \left(\sum_{i\in[n]} p(x_i)\gamma(\delta_{x_i})\right)\delta_{\overline{x}} + \left(1 - \sum_{i\in[n]} p(x_i)\gamma(\delta_{x_i})\right)\delta_{\underline{x}}$.

Proof. By repeated application of independence, Step 2, and definition of γ,

$$p = \sum_{i=1}^{n} p(x_i)\delta_{x_i} \sim \sum_{i=1}^{n} p(x_i)\left(\gamma(\delta_{x_i})\delta_{\overline{x}} + (1-\gamma(\delta_{x_i}))\delta_{\underline{x}}\right)$$
$$= \sum_{i=1}^{n} p(x_i)\left(\gamma(\delta_{x_i})\right)\delta_{\overline{x}} + \sum_{i=1}^{n} p(x_i)\left((1-\gamma(\delta_{x_i}))\right)\delta_{\underline{x}}$$

□

Step 7. Take any $p, p' \in \Delta(X)$. $p \succsim p' \iff \mathbb{E}_p[u] \geq \mathbb{E}_{p'}[u]$.

Proof. By Step 4 and Step 5, $\gamma(p)\delta_{\overline{x}} + (1-\gamma(p))\delta_{\underline{x}} \sim p \succsim p' \sim \gamma(p')\delta_{\overline{x}} + (1-\gamma(p'))\delta_{\underline{x}}$, if and only if $\gamma(p) \geq \gamma(p')$. By Step 5 and Step 6, it must be that $\mathbb{E}_p[\gamma] = \sum_{i\in[n]} p(x_i)\gamma(\delta_{x_i}) = \gamma(p)$. By definition, $\mathbb{E}_p[u] = \mathbb{E}_p[\gamma]$. □

For (ii), take u as defined in (i) and let v be some other EU representation of $\succsim$.
Note that for any $p \in \Delta(X)$, it must be that $v(\overline{x}) \geq \mathbb{E}_p[v] \geq v(\underline{x})$. Therefore, define $\phi(p)$ as the unique number such that $\phi(p)v(\overline{x}) + (1-\phi(p))v(\underline{x}) = \mathbb{E}_p[v]$.
As

$$\phi(p)v(\overline{x}) + (1-\phi(p))v(\underline{x}) = \mathbb{E}_{\phi(p)\delta_{\overline{x}}+(1-\phi(p))\delta_{\underline{p}}}[v],$$

we have that

$$\phi(p)\delta_{\overline{x}} + (1-\phi(p))\delta_{\underline{x}} \sim p \sim \gamma(p)\delta_{\overline{x}} + (1-\gamma(p))\delta_{\underline{x}}.$$

By Step 5, $\gamma(p) = \phi(p)$. Hence, $u = \frac{v - v(\underline{x})}{v(\overline{x}) - v(\underline{x})}$. □

The theorem above implies that a expected utility representation is unique up to positive affine transformations. This implies that u has a cardinal interpretation. But note that this is just the same as many structural properties of preferences we have seen: while strict monotone transformations of the utility representation are also representing the same preferences, they need not preserve additive separability or even continuity.

Exercise 2. *One standard model of risk preferences on lotteries over money is to assume that people consider two moments, the expectation of the lottery,* $\mathbb{E}_p[x]$*, and its variance,* $\mathbb{V}_p[x]$.

(i) Show that a function $U(p) = \mathbb{E}_p[x] - \mathbb{V}_p[x]/4$ *induces a preference relation that is not consistent with both independence and vNM continuity.*

(ii) Show that a utility function $U(p) = \mathbb{E}_p[x] - \mathbb{E}_p[x]^2 - \mathbb{V}_p[x]$ *induces a preference relation is consistent with both independence and vNM continuity.*

Exercise 3. *Suppose* $X = \mathbb{R}^2$ *and the consumer with income* $w > 0$ *has a von Neumann–Morgenstern utility index* $u(x)$ *where* u *is strictly increasing. Suppose that* $p' = (p'_1, p'_2) = (1,3)$ *and* $p'' =$

$(p''_1, p''_2) = (3,1)$ *denote two price vectors for* x*. Price* p' *realises with probability* $\alpha \in (0,1)$ *and price* p'' *realises with complementary probability* $1-\alpha$*; we will denote this by writing* $p \sim \alpha$*.*

There are two regimes. In regime 1, the consumer observes the price and then makes their consumption decisions, yielding an ex-ante utility of

$$\mathbb{E}_{p\sim\alpha} \max_{x\in B(p,w)} u(x).$$

In regime 2, the price is given by $\mathbb{E}_\alpha[p] = \alpha p' + (1-\alpha)p'' =: q$*, and their ex-ante utility is*

$$\max_{x\in B(q,w)} u(x).$$

(i) Suppose that $u(x) := f(x_1+x_2)$ *for some strictly increasing function* f*. Show that the consumer attains a higher ex-ante utility in regime 1.*

(ii) Is it true that for any strictly increasing, convex u*, the consumer attains a higher ex-ante utility in regime 1?*

4. Concluding Remarks

4.1. Compound Lotteries

As promised a brief discussion on compound lotteries or lotteries over lotteries. First, what is a lottery over lotteries? Take again the two lotteries we used in our initial example: p that assigns equal probability to £5 and -£5, and p' gives probability 1/2 to both £5,000 and -£5,000. A compound lottery ℓ is for instance a lottery that gives you p with probability 1/2 and p' with complementary probability. This *is not* the same as the mixture of $p'' = 1/2p + 1/2p''$, which gives you -£5, £5, -£5,000, and £5,000 all with probability 1/4; p'' is a *reduction* of ℓ and, in fact, you may value them differently. Segal (1990) provides a discussion on how you can have EU representations for preferences on $\Delta(X)$ and $\Delta(\Delta(X))$ that treat the compound lottery and the reduced lottery differently — unless the compount lottery is degenerate, i.e., assigns probability one to a specific $p \in \Delta(X)$, in which case, one would argue, there is nothing to reduce.

4.2. Issues with Expected Utility

Over lunch, during a colloquium in Paris on choice under risk,[4] sometime between 12 and 17 May 1952, Maurice Allais arguing that EU was not a good descriptive theory asked J. Leonard Savage (who we will encounter later on) the following question:

1. Which of the following two gambles do you prefer?

[4]Which included some very famous people in the discipline, such as Kenneth Arrow, Bruno de Finetti, Milton Friedman, Ragnar Frisch, Jacob Marschak, besides the two main characters in the story.

a) £2 million wp 1; or

b) £2 million wp .89; £10 million wp .10; nothing wp .01.

Savage readily answer a). Allais had then a follow-up question:

2. Which of the following two gambles do you prefer?

A) nothing wp .89; £2 million wp .11; or

B) nothing wp .90; £10 million wp .10.

To which Savage replied B). Allais then told him that his choices could not be rationalised by EU. This became known as the Allais (1953) paradox, and the evidence supports that most people make the same choices.

Exercise 4. *Show that if a person chooses a) and B) or b) and A), then their behaviour cannot be rationalised by EU. That is, if $\succsim$ are such that a) $\succ$ (resp. $\prec$) b) and B) $\succ$ (resp. $\prec$) A), then $\succsim$ cannot admit a EU representation. Which property is this violating?*

Should we just throw away the model? No. There are two reasons why we should not do that. One is if you — like Savage[5] — take a *normative* instead of descriptive stance and believe that this is a rational way to behave. In many respects, theory is also meant to provide advice on how to act (as does engineering on how to build a bridge).

Even if your inclinations are towards a more descriptive approach to modeling behaviour (as are my own), this still does not mean you should throw away the model. All models will be wrong, as they are just that, simplified descriptions. In many domains, expected utility maximisation provides a good enough approximation to describing behaviour. For that it is important to understand the conditions under which it performs well and when it fails, so that we can improve on it.

There are two plausible explanations for the Allais paradox. One is that in question 1. there is the possibility of getting something good for sure, the so-called *certainty effect*. The other is that b) has the possibility of getting nothing with positive probability and there may be a natural aversion to getting nothing. These are naturally related and you cannot disentangle them from this question alone, you need to go beyond that.

To conclude, let's point out three possible ways (out of many) that extend expected utility and accommodate Savage's intuitive choices: One is **rank-dependent expected utility** (popularised by cumulative prospect theory) (Quiggin, 1982), in which the small probabilities of the worst events loom larger than they are. A second one is **cautious expected utility** (Cerreia-Vioglio et al., 2015), which uses the following relaxion of independence: $\forall p, p' \in \Delta(X)$, $x \in X$,

[5] Savage later replied that he had acted *irrationally* and that he still thought that the properties were good characterisations of rational behaviour (Heukelom, 2015).

and $\alpha \in [0,1]$, if $p \succsim \delta_x$, then $\alpha p + (1-\alpha)p' \succsim \alpha\delta_x + (1-\alpha)p'$. A third way — **ordered reference dependent choice** (Lim, 2021) — focuses on the fact that choices depend on context: in this case, on having both a sure-thing and the possibility of gaining nothing.

5. Further Reading

Standard References: Mas-Colell et al. (1995, Chapters 6A-B), Rubinstein (2018, Chapter 7), Kreps (2012, Chapters 5), Kreps (1988, Chapters 4, 5) (advanced).

Related questions/topics: Most of the developments related to expected utility do one of three things: (1) obtain new representations to accommodate observed deviations from expected utility — some of these were cited above — by weakening the assumptions; (2) test these new models in the lab; or (3) use these new models as plug-ins to explain new phenomena. For instance, there are papers discussing the separation of risk and time preferences based on relaxing expected utility, on how to obtain loss aversion from relaxed models such as cautious expected utility, and a growing literature on motivated beliefs and wishful thinking.

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