ECON0106: **Microeconomics**

7. Stochastic Orders[*]

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1. Overview

In the previous lectures we considered how individuals evaluate distributions, e.g. of stock returns or lottery tickets prizes. We modeled their preferences of distributions and derived properties of the expected utility representation of a particular agent based on how they ranked them. In this lecture we take a different approach: we want to know how to rank distributions in an unambiguous manner among groups of individuals.

First, we look at a ranking on distributions with which *every* expected utility maximiser would agree. While this is quite a strong requirement, we obtain a simple characterisation based on how the (cumulative) distributions compare. We discuss the properties that this ordering has and a useful refinement.

Then, we look into riskiness. The idea is to have a well-grounded notion of what it means for a distribution to be riskier than another. Our strategy will be to require that *every* risk-averse expected utility maximiser would agree on which distribution is riskier. Again, this turns out to also have a simple characterisation.

Finally, we briefly discuss a recent result on how background risks can affect the ranking of distributions.

2. First-Order Stochastic Dominance

Our first ranking on the space of distributions requires every expected utility maximiser to agree.

Let $\mathscr{F}$ denote the set of all distributions on $X \subseteq \mathbb{R}$.

Definition 1. A distribution F **first-order stochastically dominates** (FOSD) a distribution G, denoted by $F \geq_{FOSD} G$ if, for all nondecreasing functions $u : X \to \mathbb{R}$, $\mathbb{E}_F[u] \geq \mathbb{E}_G[u]$.

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This means that every expected utility maximiser with increasing Bernoulli utility would weakly prefer F to G. This is quite a strong requirement. The following theorem provides a simple characterisation:

Theorem 1. *For any distributions F, G on $\mathbb{R}$, $F \geq_{FOSD} G$ if and only if, $\forall x \in X$, $F(x) \leq G(x)$.*

The first, more restrictive, version of this theorem first appeared in Hadar and Russell (1969).

Proof. $\implies$: For any $a \in X$, define $u_a(x) := \mathbf{1}_{\{x \geq a\}}$, where $\mathbf{1}_A$ is the indicator function, taking the value 1 if A is true and 0 if otherwise. Note that u_a is nondecreasing. Then,

$$F \geq_{FOSD} G \implies \mathbb{E}_F[u_a] \geq \mathbb{E}_G[u_a] \iff \int_X u_a(x)dF(x) \geq \int_X u_a(x)dG(x)$$
$$\iff \int_{x \geq a} 1dF(x) \geq \int_{x \geq a} 1dG(x), \qquad \forall a \in X$$
$$\iff 1 - F(a) \geq 1 - G(a) \iff F(a) \leq G(a), \qquad \forall a \in X.$$

$\impliedby$: For this part we are going to make use of a result in statistics called the inverse transform method. For a cumulative distribution F of a real-valued random variable, define the **generalised inverse** — also called a **quantile function** — $Q_F(\tau) := \min\{x \in \mathbb{R} \mid F(x) \geq \tau\}$, for every $\tau \in (0,1)$.[1]

The next result is also very useful in statistics, to simulate random variables given by difficult expressions:

Proposition 1. ***(Inverse Transform Method)*** *Let F be the cumulative distribution of a real-valued random variable X. Then, X has the same distribution as $Q_F(U)$, $X \stackrel{d}{=} Q_F(U)$, where U is uniformly distributed in $(0,1)$.*

The inverse transform method gives then a way to represent the distribution of X through a transformation of a standard uniformly distributed random variable. This is very convenient computationally as we know how to efficiently simulate uniformly distributed random variables. As we will see, this transformation is also helpful from a theoretical standpoint.[2] First, let's prove Proposition 1:

Proof. We want to show that $\mathbb{P}(Q_F(U) \leq x) = F(x)$. First note that Q_F is nondecreasing: As F is nondecreasing, $\forall \tau' \geq \tau$, $\{x \in \mathbb{R} \mid F(x) \geq \tau'\} \subseteq \{x \in \mathbb{R} \mid F(x) \geq \tau\} \implies Q_F(\tau) \leq Q_F(\tau')$.

Now take any $\tau \in (0,1)$ and x such that $\tau < F(x)$.

$$\tau < F(x) \implies Q_F(\tau) \leq Q_F(F(x)) \leq x$$

[1] Why min and not inf? Because, as F is nondecreasing and right-continuous with left-limits, it is upper semi-continuous, and for any τ, $\{x \in \mathbb{R} \mid F(x) \geq \tau\}$ is closed and therefore contains its infimum.

[2] Another implication of Proposition 1 is that if $X \sim F$, with F continuous, then $F(X) \sim U(0,1)$.

where the last inequality is due to $\tau < F(x) \implies x \in \{y \in \mathbb{R} \mid F(y) \geq \tau\}$ and, by definition, $Q_F(F(x)) \leq x$.

As we have that $Q_F(\tau) \leq x$ implies $\tau \leq F(x)$, we can order the following three events, recalling that U is uniformly distributed on $(0,1)$;

$$
\begin{aligned}
&\{U < F(x)\} \subseteq \{Q_F(U) \leq x\} \subseteq \{U \leq F(x)\} \\
\iff &\mathbb{P}(U < F(x)) \leq \mathbb{P}(Q_F(U) \leq x) \leq \mathbb{P}(U \leq F(x)) \\
\iff &F(x) \leq \mathbb{P}(Q_F(U) \leq x) \leq F(x).
\end{aligned}
$$

□

Let's finalise our proof of Theorem 1 by showing that $F(x) \leq G(x), \forall x \in X \implies F \geq_{FOSD} G$. Define Q_F and Q_G as the quantile functions of F and G. Then,

$$
\begin{aligned}
F(x) \leq G(x), \forall x \in X &\implies (F(x) \geq \tau \implies G(x) \geq \tau) \\
&\implies \{x \in X \mid F(x) \geq \tau\} \subseteq \{x \in X \mid G(x) \geq \tau\} \\
&\implies Q_F(\tau) \geq Q_G(\tau).
\end{aligned}
$$

As then we finally get

$$
\begin{aligned}
F(x) \leq G(x), \forall x \in X &\implies Q_F(z) \geq Q_G(z), \forall z \in (0,1) \\
&\implies u(Q_F(z)) \geq u(Q_G(z)), \forall z \in (0,1) && \text{as } u \text{ is nondecreasing} \\
&\implies \int_{[0,1]} u(Q_F(z))dz \geq \int_{[0,1]} u(Q_G(z))dz \\
&\iff \int_X u(x)dF(x) \geq \int_X u(x)dG(x) && \text{by inverse transform sampling} \\
&\iff \mathbb{E}_F[u] \geq \mathbb{E}_G[u].
\end{aligned}
$$

□

Exercise 1. *Consider $\geq_{FOSD}$ on $\Delta([0,1])$.*

1. *Prove or disprove: $\geq_{FOSD}$ is (i) reflexive; (ii) transitive; (iii) antisymmetric; (iv) complete.*
2. *In light of 1, how would you classify $(\Delta([0,1]), \geq_{FOSD})$?*
3. *Is $(\Delta([0,1]), \geq_{FOSD})$ a lattice? Is it a complete lattice?*

Exercise 2.

(i) Let $F, G, \hat{F}, \hat{G} \in \Delta(\mathbb{R})$. Show that if $\mathbb{E}_F[u] \geq \mathbb{E}_{\hat{F}}[u]$ and $\mathbb{E}_G[u] \geq \mathbb{E}_{\hat{G}}[u]$ for every nondecreasing Bernoulli utility function u, then $\alpha F + (1-\alpha)G \geq_{FOSD} \alpha\hat{F} + (1-\alpha)\hat{G}$.

(ii) For a distribution F on $\mathbb{R}$, let $G := (F + w)$, with $w > 0$. Show that $G \geq_{FOSD} F$.

Exercise 3. *Suppose an agent is selling two lottery tickets, x and y, with $x \geq_{FOSD} y$. Which one should have a higher price?*

3. Monotone Likelihood Ratio Order

One stochastic ordering that you will encounter almost surely is the monotone likelihood ratio order. For this section, we will restrict attention to distributions that either (i) admit a density or (ii) have discrete support. If the F is a distribution satisfying (i) f will denote its density, whereas if it satisfies (ii) we use f to denote its probability mass function.

Here's the definition of the monotone likelihood ratio order:

Definition 2. Let F,G two distributions on $\mathbb{R}$ and suppose that they (i) either both admit a density, or (ii) both have discrete support. F **monotone likelihood ratio** dominates G ($F \geq_{MLR} G$) if $f(x)/g(x)$ is nondecreasing in x.

One extremely convenient property of $\geq_{MLR}$ is that it is not only a partial order, but also a coarsening of $\geq_{FOSD}$ within this class of distributions:[3]

Proposition 2. *Let F,G two distributions on $\mathbb{R}$ and suppose that they (i) either both admit a density, or (ii) both have discrete support. If $F \geq_{MLR} G$, then $F \geq_{FOSD} G$.*

Proof. $f(x)g(y) \geq f(y)g(x)\,\forall x \geq y \implies f(x)G(x) - F(x)g(x) \geq 0, (1-F(x))g(x) - f(x)(1-G(x)) \geq 0\,\forall x$. As $f(x)G(x) - F(x)g(x) \geq 0 \implies \frac{f(x)}{g(x)} \geq \frac{F(x)}{G(x)}$ and $(1-F(x))g(x) - f(x)(1-G(x)) \geq 0 \implies \frac{1-F(x)}{1-G(x)} \geq \frac{f(x)}{g(x)}$, we obtain $G(x) \geq F(x)$ for all x. □

One of the reasons for why this is such a convenient ordering is that it pairs very well with Bayesian updating (i.e. Bayes' rule).

Exercise 4. *Suppose a coin toss flips heads ($x = 1$) with probability $\theta \in [0,1]$, and tails ($x = 0$) with complementary probability. Due to machine impression, θ is distributed according to a distribution F with density $f > 0$. You know f and want to estimate θ.*

(i) Show that for any sequences $x_1,\dots,x_m$, $x'_1,\dots,x'_n$ such that $n \geq m$ and $\sum_i x_i \geq \sum_i x'_i$, you have $\theta|x_1,\dots,x_m \geq_{MLR} \theta|x'_1,\dots,x'_n$. Conclude that $\mathbb{E}_F[\theta|x_1,\dots,x_m] \geq \mathbb{E}_F[\theta|x'_1,\dots,x'_n]$.

(ii) Now suppose that there is another machine that produces coins, but with a different impression: $\theta \sim G$, where $g := G' > 0$. Show that if $f \geq_{MLR} g$, then, for any sequence of coin tosses $x_1,\dots,x_m$, $f \mid x_1,\dots,x_m \geq_{MLR} g \mid x_1,\dots,x_m$. Conclude that $\mathbb{E}_F[\theta|x_1,\dots,x_m] \geq \mathbb{E}_G[\theta|x_1,\dots,x_m]$.

4. Second-Order Stochastic Dominance

Since we observe that individuals are typically risk-averse, it may be useful to know the minimal requirements under which a lottery is preferred to another for any risk-averse expected utility maximiser. This provides us also with not only natural but also a sharper definition of

[3]Some pairs of distributions can be compared according to $\geq_{FOSD}$ but not according to $\geq_{MLR}$, i.e. $\geq_{MLR} \subseteq \geq_{FOSD}$.

what it means for a lottery to be riskier than another.

Definition 3. A distribution F **second-order stochastically dominates** (SOSD) a distribution G, denoted by $F \geq_{SOSD} G$ if $\mathbb{E}_F[u] - \mathbb{E}_G[u] \geq 0$ for all nondecreasing, concave functions $u : \mathbb{R} \to \mathbb{R}$, such that $\mathbb{E}_F[u] - \mathbb{E}_G[u]$ is well-defined and $\int_{-\infty}^{0} u(x)dF(x), \int_{-\infty}^{0} u(x)dG(x) > -\infty$.

If we restrict F and G to have bounded support, $\int_{-\infty}^{0} u(x)dF(x), \int_{-\infty}^{0} u(x)dG(x) > -\infty$ is automatically satistied.

From the definitions, it should be immediate that $F \geq_{FOSD} G \implies F \geq_{SOSD} G$. That is, $\geq_{SOSD}$ is finer than $\geq_{FOSD}$ as it allows us to compare the same elements and more ($\geq_{FOSD} \subseteq \geq_{SOSD}$). The next theorem fully characterises second-order stochastic dominance from the properties of the distributions alone:

Theorem 2. *For any distributions F, G on $\mathbb{R}$, $F \geq_{SOSD} G$ if and only if, $\forall x \in X$, $\int_{-\infty}^{x} F(s)ds \leq \int_{-\infty}^{x} G(s)ds$.*

This result has had a troubled history; we follow the statement in Tesfatsion (1976).[4] Given that the proof of the statement in such generality is quite daunting, we will prove the theorem for distributions F, G with bounded support.

Proof. First, let us recall that, from integration by parts, $\int_a^b u(x)dF(x) = F(b)u(b) - F(a)u(a) - \int_a^b F(x)du(x)$. As we are assuming that F, G have bounded support, let $\overline{x}, \underline{x}$ be such that $F(\underline{x}) = G(\underline{x}) = 0$ and $F(\overline{x}) = G(\overline{x}) = 1$ and we assume u is defined on $(\underline{x} - \epsilon, \overline{x} + \epsilon)$, for some $\epsilon > 0$.

$\implies$: Let $u_a(x) = \mathbf{1}_{x \leq a}(x - a)$, a nondecreasing and concave function. From integration by parts, we have $\int_{\underline{x}}^{a} u_a(x)dF(x) - \int_{\underline{x}}^{a} u_a(x)dG(x) = (F(a) - G(a))(a - a) - (F(\underline{x}) - G(\underline{x}))u_a(\underline{x}) + \int_{\underline{x}}^{a}(G(x) - F(x))dx = \int_{\underline{x}}^{a}(G(x) - F(x))$. Then,

$$\begin{aligned}
\mathbb{E}_F[u_a] - \mathbb{E}_G[u_a] \geq 0, \quad \forall a &\iff \int_{x \leq a} u_a(x)dF(x) \geq \int_{x \leq a} u_a(x)dG(x), \quad \forall a \\
&\iff \int_{x \leq a} u_a(x)dF(x) - \int_{x \leq a} u_a(x)dG(x) \geq 0, \quad \forall a \\
&\iff \int_{x \leq a} (G(x) - F(x))dx \geq 0, \quad \forall a \\
&\iff \int_{\underline{x}}^{a} F(x)dx \leq \int_{\underline{x}}^{a} G(x)dx, \quad \forall a.
\end{aligned}$$

$\impliedby$: Let us construct a linear interpolation of any concave nondecreasing u on $[\underline{x}, \overline{x}]$.

For any $n \in \mathbb{N}$ let $x_i^n := \underline{x} + \frac{i}{n}(\overline{x} - \underline{x})$ for $i = 0, \ldots, n$. The set $\{x_i^n\}_{i=0}^n$ is an evenly spaced grid on

[4] In case you find a weaker statement with a correct proof, do let me know; our restrictions are hidden in how we defined $\geq_{SOSD}$. Early versions of this theorem appeared in Hadar and Russell (1969) and Hanoch and Levy (1969). The first one imposed excessively restrictive assumptions: finite support and strictly increasing utility. The second imposed no restrictions — not even our condition — but is incorrect; a corrected version was given by Tesfatsion (1976), which we follow in our statement.

$[\underline{x},\overline{x}]$, where $x_{i+1}^n - x_i^n = \frac{1}{n}(\overline{x} - \underline{x})$.

Now we want to construct u^n defined on $[\underline{x},\overline{x}]$ such that $u^n(x_i^n) = u(x_i^n)$, that is, that touches u at each point in the grid, and is a linear interpolation of u, which means that it will also be nondecreasing and concave. To do this, for $i = 0,...,n-1$ define $c_i^n := \frac{u(x_{i+1}^n)-u(x_i^n)}{x_{i+1}^n - x_i^n}$, which gives the slope of the line that connects $u(x_i^n)$ to $u(x_{i+1}^n)$. Let $c_n^n := 0$. As u is nondecreasing, we must have that $c_i^n \geq 0$. Furthermore, as u is concave, we have that c_i^n is nonincreasing in i.

Exercise 5. *Prove that c_i^n is nonnegative and nonincreasing in i.*

For any $x \in [\underline{x},\overline{x}]$, by construction, $\exists i = 0,...,n-1$ such that $x \in [x_i^n, x_{i+1}^n]$, and then we define $u^n(x) := u(x_i^n) + c_i^n(x - x_i^n)$.[5] Clearly, $u^n(x_i^n) = u(x_i^n)$ for every $i = 0,...,n$ and $u(x) - u^n(x) \geq 0$.

Exercise 6. *Prove that*

(i) u^n is concave. (Hint: Try showing that at any point $x \in [\underline{x},\overline{x}]$ the supergradient of u^n is non-empty.)

(ii) For any $x \in [x_i^n, x_{i+1}^n]$ and $y \in [x_k^n, x_{k+1}^n]$, then $u^n(y) + c_i^n(x-y) \leq u^n(x) \leq u^n(y) + c_k^n(x-y)$.

(iii) For any $x \in [x_i^n, x_{i+1}^n]$, $|u(x) - u(x_i^n)| \leq |u(x_1^n) - u(\underline{x})|$.

Given (iii) from the exercise above, we have that

$$|u(x) - u(x_i^n)| \leq u(x_1^n) - u(\underline{x}).$$

As u is defined on an open interval and real-valued concave functions on an open interval are continuous, then $\lim_{n\to\infty} \sup_{x\in[\underline{x},\overline{x}]} |u^n(x) - u(x)| \leq \lim_{n\to\infty} u(x_1^n) - u(\underline{x}) = 0$, and we have that u^n converges uniformly to u.

Exercise 7. *Recall our definition $u_a(x) := \mathbf{1}_{x\leq a}(x-a)$. Let $d_n^n := c_n^n$ and, for $i = 0,1,...,n-1$, let $d_i^n := c_i^n - c_{i+1}^n$. Define $\tilde{u}^n(x) := u(\overline{x}) + \sum_{i=1}^n d_{i-1}^n u_{x_i^n}(x)$.*

(i) Prove that $\tilde{u}^n = u^n$. (Hint: Either you show it by brute-force algebra; or you show that (a) the $\tilde{u}^n = u^n$ at the points in our grid $\{x_i^n\}_{i=0,1,...,n}$, (b) show that $\forall x \in [x_i^n, x_{i+1}^n]$, $\tilde{u}^n(x) - \tilde{u}^n(x_i^n) = u^n(x) - u^n(x_i^n)$.)

(ii) Use (i) to prove that if $\mathbb{E}_F[u_a] \geq \mathbb{E}_G[u_a]$, for every $a \in [\underline{x},\overline{x}]$, then $\mathbb{E}_F[u^n] \geq \mathbb{E}_G[u^n]$, for every n.

Then,

$$\int_{\underline{x}}^a F(x)dx \leq \int_{\underline{x}}^a G(x)dx, \quad \forall a \implies \mathbb{E}_F[u^n] \geq \mathbb{E}_G[u^n], \quad \forall n.$$

As u^n converges uniformly and is integrable, then

$$0 \leq \lim_{n\to\infty} \mathbb{E}_F[u^n] - \mathbb{E}_G[u^n] = \mathbb{E}_F[u] - \mathbb{E}_G[u].$$

[5] This is equivalent to defining $u^n(x) := \mathbf{1}_{x=\underline{x}} u(\underline{x}) + \sum_{i\in\{0,...,n-1\}} \mathbf{1}_{x\in(x_i^n, x_i^n]}[u(x_i^n) + c_i^n(x - x_i^n)]$.

□

A more restrictive notion of a gamble F being “riskier” than another gamble G requires that both F and G have the same mean but G has higher variance.

Definition 4. Let F, G be distributions on $\mathbb{R}$. G is a **mean-preserving spread** of F if there are random variables X, Y, and ϵ, such that $Y \overset{d}{=} X + \epsilon$, $X \sim F$, $Y \sim G$, and $\mathbb{E}[\epsilon \mid X] = 0$.

Exercise 8. *1. Let $\geq_{MPS}$ be such that $G \geq_{MPS} F$ if G is a mean-preserving spread of F. Prove that $G \geq_{MPS} F \implies F \geq_{SOSD} G$, but that the converse is not true in general.*

2. Show that if $F \geq_{SOSD} G$, then $\mathbb{E}_F[x] \geq \mathbb{E}_G[x]$.

3. Show that if $G \geq_{MPS} F$, then $\mathbb{E}_F[x] = \mathbb{E}_G[x]$ and $\mathbb{V}_F[x] \leq \mathbb{V}_G[x]$.

4. Prove $F \geq_{FOSD} G \implies F \geq_{SOSD} G$, but that the converse is not true in general.

5. Show that $\geq_{SOSD}$ and $\geq_{MPS}$ are partial orders.

4.1. Second-Order Stochastic Dominance in $\mathbb{R}^n$ (*)

The discussion above extends to more general spaces;[6] we focus on extending our characterisation of SOSD to $\mathbb{R}^n$.

Definition 5. Let F and G be distributions on $\mathbb{R}^n$. We say that F **second-order stochastically dominates** G ($F \geq_{SOSD} G$) iff $\mathbb{E}_F[u] \geq \mathbb{E}_G[u]$ for all nondecreasing concave $u : \mathbb{R}^n \to \mathbb{R}$, whenever both expectations exist.

The key result in this section, which we state for reference, is the following:

Theorem 3. *(Strassen’s (1965) Theorem) Let F and G be distributions on $\mathbb{R}^n$ with bounded support. Then, $F \geq_{SOSD} G$ if and only if there are $X \sim F$ and $Y \sim G$ such that $X \geq \mathbb{E}[Y \mid X]$ a.s.*

In short, what this theorem is giving is a way to define a joint distribution $H(x, y)$ such that the marginals over x and y equal F and G, respectively, and $\int_{\mathbb{R}^n} yH(x, y)dy \leq x$.[7]

We can also adjust the definition of mean-preserving spreads as expected and obtain a useful corollary:

Corollary 1. *Let F and G be distributions on $\mathbb{R}^n$ with bounded support. G is a mean-preserving spread of F if and only if $F \geq_{SOSD} G$ and $\mathbb{E}_F[x] = \mathbb{E}_G[x]$.*

[6] For a reference, see Müller and Stoyan (2002).
[7] Strassen (1965) proves far more general results; see Müller and Stoyan (2002, Theorem 2.6.8) for a recent proof.

5. Background Risks

When considering investing in stock X or Y, professional traders typically consider only their expected return, that is, whether $\mathbb{E}[X] > \mathbb{E}[Y]$, and potentially not so much the associated risk, captured, for instance, by $\mathbb{V}[X], \mathbb{V}[Y]$. While one may consider that they have risk attitudes that are very particular — risk aversion is a common finding, both empirical and experimentally — another possible explanation is that they are considering the existence of background risks. Stock X and Y are not the only stocks that traders invest in, and large traders are likely to have large amounts of money invested in a number of different stocks that add to background risk on their portfolio.

A recent paper by Pomatto et al. (2020) explores the connection between background risks and stochastic orders. In short, the result says that there are (independent!) background risks large enough that, when considered, can dwarf any riskiness considerations and make even the most risk-averse person to simply go with the gamble that yields the highest expected value. An abbreviated statement goes as follows:

Theorem 4. *(Pomatto et al., 2020) Let X and Y be random variables with finite variance.*

(i) If $\mathbb{E}[X] > \mathbb{E}[Y]$, then there is an independent random variable ϵ such that $X + \epsilon \geq_{FOSD} Y + \epsilon$.

(ii) If $\mathbb{E}[X] = \mathbb{E}[Y]$ and $\mathbb{V}[X] < \mathbb{V}[Y]$, then there is an independent random variable ϵ such that $X + \epsilon \geq_{SOSD} Y + \epsilon$.

6. Further Reading

Standard References: Mas-Colell et al. (1995, Chapter 6D), Rubinstein (2018, Chapter 8), Kreps (2012, Chapter 6).

Related questions/topics: Stochastic orders establish natural comparisons across distributions. Kleiner, Moldovanu, and Strack (2021) derive a number of useful properties related to mean-preserving spreads and use them to study questions related to auctions, delegation, and decision-making under uncertainty (among others). These show up again and again when studying questions related to information — e.g. pricing data, information disclosure, designing tests (from financial stress tests to exams), etc.

There have been some recent developments expanding our understanding of these orders. Beyond the the mentioned paper on background risks, I'll refer also Lehrer and Wang (2020), who introduce the notion of strong stochastic dominance and discuss applications to Bayesian learning and asset pricing, and Meyer and Strulovici (2012), who study orders of interdependence — useful for finance (valuing portfolios), empirical work (inputing data), and measuring the degree of alignment of preferences in decision-making in groups (e.g. voting).

Why do we like orderings and monotonicity? Because we can (try to) use them to obtain monotone comparative statics results. This is what Jensen (2018) does, but in the space of distributions (of income, of information, of ability, etc).

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