ECON0106: **Microeconomics**

Problem Set 2

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Question 1. 2. Structural Properties of Preferences and Utility Representations: Exercise 1

Question 2. In many circumstances, individuals do not consider all available alternatives, be it because they follow some heuristic, or simply because comparing all of them is just infeasible (anyone who does online shopping would agree). We will consider a model in which our decision maker's choices are explained by maximizing utility but consider just a subset of all the available alternatives. We will denote by X the set of all alternatives and, for simplicity, we will consider choice functions (instead of correspondences) $C : 2^X \to X$, that is, $C(A) \in A$ for any nonempty $A \subseteq X$.[1] We will throughout this question assume that X is **finite**.

Definition 1. A choice function $C : 2^X \to X$ is said admit a choice with consideration sets representation if there is (i) a utility function $u : X \to \mathbb{R}$ and (ii) a consideration set correspondence $\Gamma : 2^X \to 2^X$ such that for any nonempty $A \subseteq X$, $\emptyset \neq \Gamma(A) \subseteq A$ and $C(A) = \arg\max_{x \in \Gamma(A)} u(x)$.

This exercise is going to guide you through a useful model of choice with consideration sets and its implications for choices.

Q2.(i) The first question is: does this model have any bite? Show that, without any further restrictions on Γ, any choice function can be represented as in Definition 1.

Q2.(ii) We say that Γ satisfies **property A** if $\Gamma(A) = \Gamma(A \setminus \{x\})$ for any $x \notin \Gamma(A)$.
In a nutshell, if $\Gamma(A)$ determines which elements of A the decision maker is considering (paying attention to), property A says that if we remove an element in A that the decision maker is not considering, it won't affect the decision maker's consideration set.
Now consider the following heuristics:

(a) The decision maker is choosing between fridges, and, in any choice set, considers only the 3 fridges that are most energy-efficient (assume there are no ties).
(b) The decision maker is considering offers from different PhD programs according to the quality of their theory faculty, the stipend they receive, and the city's cultural offer. In any

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[1] The arguments do extend to choice correspondences easily!

choice set, the decision maker considers the best alternatives in each category (assume there are no ties).

(c) The decision maker is choosing between laptops, and considers a laptop only if it is at or below the median price in the choice set (no ties in the prices).

For which of the above heuristics does the method of constructing the consideration set satisfy property A? Prove your claims.

Q2.(iii) Construct a choice dataset $(A_n, C(A_n))_{n\in[N]}$ that cannot be explained by choice with consideration sets satisfying property A.

Q2.(iv) Show that, under a model of choice with consideration sets satisfying property A, if $x = C(A)$ and $y \in A$, then it could be the case that $u(y) > u(x)$.

Define a binary relation P on X as follows: xPy if for some choice set A such that $x, y \in A$, $x \neq y$, then $x = C(A) \neq C(A \setminus \{y\})$. We can think about P as showing that x was chosen while y was considered.

Show that, under a model of choice with consideration sets satisfying property A, if xPy, then $u(x) > u(y)$.

Q2.(v) We say that the choice function C satisfies **property B** if, for any nonempty set $S \subseteq X$, there exists $x^* \in S$ such that for any $T \subseteq X$ that includes x^*, $C(T) = x^*$ if $C(T) \in S$ and $C(T) \neq C(T \setminus \{x^*\})$.

Property B is satisfied if and only if P is acyclic.[2]

The rest of this exercise is devoted to proving the following:

Theorem 1. *C satisfies property B if and only if C admits a choice with consideration set representation where Γ satisfies property A.*

In other words, the exact behavioral properties of our model.

Q2.(vi) First, let's show that: if C admits a choice with consideration set representation where Γ satisfies property A, then C satisfies property B.

Q2.(vii) A quick aside: show that Sen's α implies property B, but that the converse is not true in general.

Now let's take care of the only if part. As we have shown that property B is equivalent to P being acyclic and as we said that if xPy, then y must've been paid attention to, we'll leverage this intuition to construct our consideration set.

Q2.(viii) For any nonempty $A \in 2^X$, define $\Gamma(A) := C(A) \cup \{y \in A \mid C(A)Py\}$. Conclude our proof by showing that, if C satisfies property B, then C admits a choice with consideration set representation where Γ satisfies property A.

[2]That is, for any $N \in \mathbb{N}$, there is no $\{x_i\}_{i\in[N]} \subseteq X$, such that $x_i P x_{i+1}$ for $i = 1, \ldots, N-1$, and $x_N P x_1$.

Q2.(ix) What do you think of the model? What is it capturing well? What is it missing?