ECON0106: **Microeconomics**

Problem Set 7

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Question 1. Suppose there are N firms (finite N), i.e. $|I| = N \geq 2$. Each firm i chooses quantity $q_i \in \mathbb{R}_+$. Firm i's payoffs are given by their profits, $\pi_i(q_i, q_{-i}) := P(q)q_i - cq_i$, where $P(q)$ denotes the inverse demand, given by $P(q) = \max\{0, a - b(\sum_{j \in I} q_j)\}$, with $a, b, c > 0$. Solve for the set of rationalizable (pure) strategies. Comment on your findings.

Question 2. A friend doing IO is doing a complicated structural estimation that included a finite but large and complicated game. They noted (1) that each agent had a weakly dominant strategy and (2) that the game had a unique PSNE, but they didn't want to rule out any MSNE. Your friend turns to you for help: is there any other NE in the game other than the unique PSNE?

Prove or disprove the following statement: A finite normal-form game in which each player has a weakly dominant strategy and which has a unique pure-strategy Nash equilibrium, has a unique Nash equilibrium.

Question 3. Every day, many of the good people of Alterbury commute to Blackwood-upon-Avon. There are two main roads connecting Alterbury and Blackwood-upon-Avon, a northern road via Gainsboroughshire and a southern road via Dunstableford, all of which are fictitious medieval-English-sounding towns. Travel time on each of the roads depends on the number x of wagons using the road per hour, as follows:

Origin	Destination	Time
Alterbury	Gainsboroughshire	$1 + x$
Alterbury	Dunstableford	$51 + 0.1x$
Gainsboroughshire	Blackwood-upon-Avon	$51 + 0.1x$
Dunstableford	Blackwood-upon-Avon	$1 + x$

For example, the travel time between Alterbury and Gainsboroughshire is $1 + x$, where x is the number of wagons per hour using the road connecting these towns, and the travel time between Gainsboroughshire and Blackwood-upon-Avon is $51 + 0.1x$, where x is the number

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of wagons per hour using the road connecting those two towns. Each wagon driver chooses which road to take in going from Alterbury to Blackwood-upon-Avon, with the goal of reducing to a minimum the amount of travel time. Early in the morning, 60 wagons per hour set out from Alterbury to Blackwood-upon-Avon.

(i) Describe this situation as a strategic-form game, in which each driver chooses the route they will take.

(ii) What are all the Nash equilibria of this game? At these equilibria, how much time does the trip take at an early morning hour?

(iii) The King's Road Commission constructs a new road between Gainsboroughshire and Dunstableford, with travel time between these towns $10+0.1x$. This road is one way, enabling travel solely from Gainsboroughshire to Dunstableford. Find a Nash equilibrium in the new game. Under this equilibrium how much time does it take to get to Blackwood-upon-Avon from Alterbury at an early morning hour?

(iv) Does the construction of the additional road improve travel time? Comment on your findings.