Costly Information Acquisition

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Topics in Economic Theory

November 2024

Agenda

Before: Understand information and ranking information structures.

Today: Understand choice of information and implications for choice at large.

Motivation

"Information is power": Blackwell order.

... but "There's no such thing as a free lunch": Information is costly.

- Time
- Labor: need to pay people to gather the data (e.g. running surveys, parsing reviews), when not to elicit data in IC manner (e.g. econ experiments)
- Capital: labs, IT (data crunching), human capital (understanding what computer memory even is)

Today

Idea: learning about unknown state is costly.

Pay attention, process available information, gather information, etc.

Stylised model of costly info acquisition

Can design/choose costly experiment.

Static problem, know available actions.

All bundled into cost of info.

Miss out on otherobsevables: time, investment, limits to what is knowable.

Different models tailored to different problems.

Goals

Understanding properties of $P(a|\theta)$ (and V, and $(\mu|a)$), coming from $\max_\pi V(G^A(\pi,\mu),\pi)$.

Refining properties/model: (1) $V(G^A(\pi,\mu),\pi) = G^A(\pi,\mu) - K(\pi)$ vs. $V(G^A(\pi,\mu),\pi) = R(\pi)G^A(\pi,\mu)$; (2) several versions of K.

Overview

1. Costly Information Acquisition

2. Non-Separable Costs

3. Additive Costs

4. Multiplicative Costs

5. Constrained Information Acquisition

6. Relation to Stochastic Choice

7. Entropy Costs

8. Limitations of and Alternatives to Entropy Costs

9. Implications of CIA for Choice

10. Experimental Evidence

11. Applications

Overview

1. Costly Information Acquisition
   - Model
   - Value and Cost of Information
   - State-Dependent Stochastic Choice Data

2. Non-Separable Costs

3. Additive Costs

4. Multiplicative Costs

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9. Implications of CIA for Choice

Model

Main References: Caplin & Dean ('15 AER); Chambers, Liu, & Rehbeck ('20 JET)

Unknown state of the world $\theta \in \Theta$, finite

Prior prob distribution $\mu \in \Delta(\Theta)$.

Decision problem: A

Action a mapping from states to prize space Z, $a : \Theta \to Z$.

Finite decision problems:. $\mathcal{A} := \{A \subset Z^{\Theta} \mid |A| < \infty\}$

vNM utility $u : Z \to \mathbb{R}$; write $u(a, \theta) \equiv u(a(\theta))$.

Assumption: u and μ are known (more on this later).

Model

Information structure $\pi : \Theta \to \Delta(\Delta(\Theta))$ with finite support $S(\pi)$

Identify signals with posterior distributions: for $m \in S(\pi)$

$$m(\theta) = \frac{\mu(\theta)\pi(m \mid \theta)}{\sum_{\theta'} \mu(\theta')\pi(m \mid \theta')}$$

Π: set of all such info structures.

Conditional posterior distrib: $\pi(m \mid \theta)$.

Unconditional posterior distrib: $\pi(m) := \mathbb{E}_\mu[\pi(m|\theta)] = \sum_\theta \mu(\theta)\pi(m \mid \theta)$.

$\mathbb{E}$ Posterior = Prior: $\mathbb{E}_\pi[m] = \mu$

$m \sim \pi$ MPS of μ.

Implicit assumption here: two signals inducing the same posterior beliefs are treated as the same signal.

Value of Information

EU max: $U^A(\mu) := \max_{a \in A} \mathbb{E}_\mu[u(a, \theta)]$

Continuous (convex) function on $\Delta(\Theta)$

Gross value of information π in decision problem A given prior μ:

$$G^A(\pi, \mu) := \mathbb{E}_\mu \left[\mathbb{E}_{\pi(\cdot|\theta)} \left[U^A(m) \right] \right] = \sum_\theta \mu(\theta) \sum_m \pi(m \mid \theta) \max_{a \in A} \mathbb{E}_m[u(a, \theta)]$$

Write $G^A(\pi, \mu) \equiv \pi \cdot U^A$.

Will omit dependence on μ where unambiguous.

Optimal Choices given Information π

Choice function $\sigma_A : S(\pi) \to \Delta(A)$ is optimal given π if, $\forall m \in S(\pi)$

$$\text{supp}(\sigma_A(m)) \subseteq \arg\max_{a \in A} \mathbb{E}_m[u(a, \theta)]$$

Note: $G^A(\pi) = \max_{\sigma_A} \mathbb{E}_\mu \left[E_{\pi(\cdot|\theta)} \left[E_{\sigma_A}[\mathbb{E}_m[u(a, \theta)]] \right] \right]$

Cost of Information

Net value of information π in decision problem A given prior μ

$$V(\pi \cdot U^A, \pi)$$

where $V : \mathbb{R} \times \Pi \to \mathbb{R} \cup -\infty$ s.t. V strictly increasing in 1st arg (when finite).

Examples:

Additive costs: $V(\pi \cdot U^A, \pi; \mu) = G^A(\pi, \mu) - K(\pi; \mu)$.

Multiplicative costs: $V(\pi \cdot U^A, \pi; \mu) = R(\pi; \mu) G^A(\pi, \mu)$.

Combinations of the two.

Additive prior-independent costs: $V(\pi \cdot U^A, \pi; \mu) = G^A(\pi, \mu) - K(\pi)$.

(Which one nests discounting with stochastic stopping?)

Optimal Information Acquisition

$$\Pi^*(A) := \arg\max_{\pi \in \Pi} V(\pi \cdot U^A, \pi)$$

State-Dependent Stochastic Choice Data

Definition

A **state-dependent stochastic choice dataset** is

a finite collection of decision problems $\mathcal{D} \subset \mathcal{A}$; and

a set of state-dependent stochastic choice functions $P = \{P_A\}_{A \in \mathcal{D}}$ s.t. $P_A : \Theta \to \Delta(A)$.

$P_A(\theta)(a) \equiv P_A(a \mid \theta)$

Analyst can observe frequency with which each action is taken at each state:

{add egg, not}, {good, rotten} (Savage's problem)

Investment decisions, realised return.

Pricing decisions, inflation.

Overview

1. Costly Information Acquisition

2. Non-Separable Costs
   - Revealed Information Structures
   - No Improving Action Switches (NIAS)
   - Generalised Axiom of Costly Information (GACI)
   - Representation Result
   - Unobservables

3. Additive Costs

4. Multiplicative Costs

5. Constrained Information Acquisition

6. Relation to Stochastic Choice

7. Entropy Costs

8. Limitations of and Alternatives to Entropy Costs

Representation

Want "(A) *P* can be rationalised by a costly info acquisition model sat. [some properties] iff (B) *P* sat. [some properties]".

(A) has two main requirements on *P*

Choices optimal given info structure.

Info structure optimal.

Representation

Definition

Given $\mu \in \Delta(\Theta)$ and $u : Z \to \mathbb{R}$, a state-dependent stochastic choice dataset $(\mathcal{D}, P)$ admits a **costly information acquisition representation** if there are

a) $V : \mathbb{R} \times \Pi \to \mathbb{R} \cup \{-\infty\}$;

b) information structures $\{\pi_A\}_{A \in \mathcal{D}}$;

c) choice functions $\{\sigma_A\}_{A \in \mathcal{D}}$;

such that

1. **Monotonicity** of V: $\forall \pi \in \Pi, r' > r, V(r, \pi) > -\infty \implies V(r', \pi) > V(r, \pi)$ $V : \mathbb{R} \times \Pi \to \mathbb{R} \cup \{-\infty\}$;
2. **Non-triviality**: $\forall r$, $\exists \pi_r$ such that $V(r, \pi_r) > -\infty$;
3. **Choices σ_A are optimal**;
4. **Information π_A is optimal**;
5. **Data are matched**: $\forall A \in \mathcal{D}, \theta \in \Theta, a \in A$,
   $P_A(a|\theta) = \mathbb{E}_{\pi(\cdot|\theta)}[\sigma_A(a|m)] = \sum_{m \in S(\pi)} \pi(m \mid \theta)\sigma_A(a|m)$

Revealed Information Structures

First step: base representation conditions on observables.

Revealed Posterior: given $\mu, A \in \mathcal{D}, P_A \in P$, and $a \in \text{supp}(P_A)$, revealed posterior $\bar{m}^a \in \Delta(\Theta)$ is given by

$$\bar{m}^a(\theta) = \mathbb{P}(\theta \mid a \text{ is chosen from } A) = \frac{\mu(\theta) P_A(a \mid \theta)}{\sum_{\theta'} \mu(\theta') P_A(a \mid \theta')}$$

Revealed Information Structures: given $\mu, A \in \mathcal{D}, P_A \in P$, and $a \in \text{supp}(P_A)$, revealed information structure $\bar{\pi}_A$ is given by

$$\bar{\pi}_A(m \mid \theta) = \sum_{a \in \text{supp}(P_A) \mid \bar{m}^a = m} P_A(a \mid \theta)$$

inducing distrib on revealed posteriors $\bar{\pi}_A(\bar{m}^a) = \sum_\theta \mu(\theta) \bar{\pi}_A(\bar{m}^a \mid \theta)$

Idea: Suppose that $A = \{0, 1\}$, and $S(\pi) = \{m, m', m''\}$;

given m, m' chose 0; given m'' chose 1

Could do as well by bundling m, m' into a single signal m^0 at with $a = 0$ is optimal

Revealed Information Structures

Definition

π_A is **consistent** with P_A if $\exists \sigma_A : S(\pi) \to \Delta(A)$ such that

(i) Choices σ_A are optimal

$$\text{supp}(\sigma_A(m)) \subseteq \arg\max_{a \in A} \mathbb{E}_m[u(a, \theta)]\, P_A(a|\theta)$$

(ii) Data are matched

$$P_A(a|\theta) = \mathbb{E}_{\pi(\cdot|\theta)}[\sigma_A(m)] = \sum_{m \in S(\pi)} \pi(m \mid \theta)\sigma_A(m)$$

Lemma

If π_A is consistent with P_A, then it is sufficient for $\bar{\pi}_A$

Intuition: SDSC data cannot be + informative than info structure that induced it

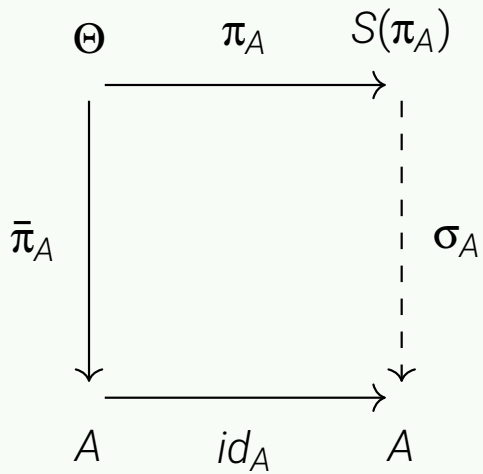
Revealed Information Structures

Lemma

If $\boldsymbol{\pi}_A$ is consistent with P_A, then it is sufficient for $\bar{\boldsymbol{\pi}}_A$

Proof Sketch

Relabel $\bar{m}^a$ as a; Then $\boldsymbol{\sigma}_A \circ \boldsymbol{\pi}_A = P_A = id_A \circ \bar{\boldsymbol{\pi}}_A = \bar{\boldsymbol{\pi}}_A$ (recall Oliveira ('18 GEB))



Revealed Information Structures

Definition

π_A is **consistent** with P_A if $\exists \sigma_A : S(\pi) \to \Delta(A)$ such that

(i) Choices σ_A are optimal

$$\text{supp}(\sigma_A(m)) \subseteq \arg\max_{a \in A} \mathbb{E}_m[u(a, \theta)]\, P_A(a|\theta)$$

(ii) Data are matched

$$P_A(a|\theta) = \mathbb{E}_{\pi(\cdot|\theta)}[\sigma_A(m)] = \sum_{m \in S(\pi)} \pi(m \mid \theta)\sigma_A(m)$$

Lemma

If π_A is consistent with P_A, then it is sufficient for $\bar{\pi}_A$

Lemma

For all $A, B \in \mathcal{D}$, if π_A is consistent with P_A, then $\pi_A \cdot U^B \geq \bar{\pi}_A \cdot U^B$ and $\pi_A \cdot U^A = \bar{\pi}_A \cdot U^A$

$\geq$ immediately from Blackwell ordering; = follows from fact that data is matched

Optimality of Actions

Note that if π_A is consistent with P_A, then so is $\bar{\pi}_A$

As $G^A(\pi_A) = G^A(\bar{\pi}_A)$, then $\sigma_A(a' \mid \bar{m}^a) := 1_{a'=a}$ is optimal given $\bar{\pi}_A$; i.e.

$$a \in \arg\max_{a' \in A} \mathbb{E}_{\bar{m}^a}[u(a', \theta)] \iff \sum_{\theta} \bar{m}^a(\theta)[u(a, \theta) - u(b, \theta)] \geq 0, \ \forall b \in A$$

$$\iff \sum_{\theta} \frac{\mu(\theta) P_A(a \mid \theta)}{\sum_{\theta'} \mu(\theta') P_A(a \mid \theta')}[u(a, \theta) - u(b, \theta)] \geq 0, \ \forall b \in A$$

$$\iff \sum_{\theta} \mu(\theta) P_A(a \mid \theta)[u(a, \theta) - u(b, \theta)] \geq 0, \ \forall b \in A$$

Definition

Given $\mu, u, (\mathcal{D}, P)$ sat. **no improving action switches** (NIAS) if for every $A \in \mathcal{D}$, and every $a \in A$ such that $P_A(a) > 0$, $\sum_{\theta} \mu(\theta) P_A(a \mid \theta)[u(a, \theta) - u(b, \theta)] \geq 0, \ \forall b \in A$

Optimality of Actions

Taking stock

Enough to get one signal for each action chosen with positive prob.

NIAS: $\sum_{\theta} \mu(\theta) P_A(a \mid \theta)[u(a, \theta) - u(b, \theta)] \geq 0$, for action a chosen wp > 0.
Required for choices to be optimal given some info structure and data to be matched.

(NIAS as an obedience constraint (BNE, BCE): player has no incentive to deviate from recommended course of action given signal realisation)

Next: implications of optimal information structure

Optimality of Information Structures

Definition

Given μ, u, $(\mathcal{D}, P)$ sat. **generalised axiom of costly information** (GACI) if for all sequences $(\bar{\pi}_{A_1}, U^{A_1}), (\bar{\pi}_{A_2}, U^{A_2}), ..., (\bar{\pi}_{A_k}, U^{A_k})$ with $A_i \in \mathcal{D}$ for which $\bar{\pi}_{A_i} \cdot U^{A_i} \leq \bar{\pi}_{A_i} \cdot U^{A_{i+1}}$ (mod k), all inequalities hold with equality.

Where does this come into play? Related to Afriat's thm! (Chambers & Echenique '16)

Lemma (Afriat's Theorem)

Let $\mathcal{D}$ be finite and for each $A, B \in \mathcal{D}$ let $u_{A,B} \in \mathbb{R}$.
Suppose that for all $A \in \mathcal{D}$, $u_{A,A} = 0$ and that for any sequence $A_1, A_2, ..., A_k$ s.t. $u_{A_i,A_{i+1}} \leq 0$ for all i (mod k), it holds that $u_{A_i,A_{i+1}} = 0$ for all i (mod k).
Then there are numbers $U_A \in \mathbb{R}$ and $\lambda_A > 0$ s.t. $\forall A, B \in \mathcal{D}$, $U_A \leq U_B + \lambda_B u_{B,A}$.

Restrictions on the Cost of Information

Would like to get implicit cost to sat. reasonable properties

$V : \mathbb{R} \times \mathbf{\Pi} \to \mathbb{R} \cup \{-\infty\}$ sat.

Weak monotonicity if π is sufficient for π' then for any $t \in \mathbb{R}$, $V(t, \pi) \leq V(t, \pi')$.
Idea: More informative info structures are more costly.

Quasiconcavity if $\forall \lambda \in [0, 1], (t, \pi), (t', \pi') \in \mathbb{R} \times \mathbf{\Pi}$, $V(t'', \pi'') \geq V(t', \pi') \wedge V(t, \pi)$, where $(t'', \pi'') = \lambda(t, \pi) + (1 - \lambda)(t', \pi')$.
Idea: Implies qconcavity in π (setting $t = \pi \cdot U$ and $t' = \pi' \cdot U$).

Normalisation if $V(0, \pi_0) = 0$, where π_0 is s.t. $\pi_0(\mu \mid \theta) = 1 \, \forall \theta$.
Idea: utility is normalised to zero when ex-ante payoff is zero and info structure is fully uninformative (posteriors = prior).

Representation Result

Theorem (Chambers, Liu, & Rehbeck, '20 JET)

Fix μ and u and let $(\mathcal{D}, P)$ be a state-dependent stochastic choice dataset. The following are equivalent:

(i) $(\mathcal{D}, P)$ sat. NIAS and GACI.

(ii) $(\mathcal{D}, P)$ admits a costly information acquisition representation.

(iii) $(\mathcal{D}, P)$ admits a costly information acquisition representation with V satisfying weak monotonicity, quasiconcavity, and normalisation.

Restrictions on V come for free; i.e. are untestable

Representation Result

Proof Sketch

- CIA rep w/ properties $\implies$ NIAS immediate from fact that if $\boldsymbol{\pi}_A \in \boldsymbol{\Pi}^*(A)$ and choices are optimal, then $\boldsymbol{\pi}_A$ is consistent with P_A $\implies$ (lemma) $\bar{\boldsymbol{\pi}}_A$ is consistent with P_A
- CIA rep w/ properties $\implies$ GACI

$V(\boldsymbol{\pi}_A \cdot U^A, \boldsymbol{\pi}_A) > -\infty$: non-triviality $\forall r, \exists \boldsymbol{\pi}_r$ s.t. $-\infty < V(r, \boldsymbol{\pi}_r)$

Let $c := \inf_{m \in \Delta(\Theta)} U^A(m)$ and $\boldsymbol{\pi}_c : -\infty < V(c, \boldsymbol{\pi}_c)$

$\implies -\infty < V(c, \boldsymbol{\pi}_c) \leq V(\boldsymbol{\pi}_c \cdot U^A, \boldsymbol{\pi}_c) \leq V(\boldsymbol{\pi}_A \cdot U^A, \boldsymbol{\pi}_A)$

Suppose $(\bar{\boldsymbol{\pi}}_{A_1}, U^{A_1}), ..., (\bar{\boldsymbol{\pi}}_{A_k}, U^{A_k})$ s.t. $\bar{\boldsymbol{\pi}}_{A_i} \cdot U^{A_i} \leq \bar{\boldsymbol{\pi}}_{A_i} \cdot U^{A_{i+1}}$ for all i (mod k)

$$V(\boldsymbol{\pi}_{A_i} \cdot U^{A_i}, \boldsymbol{\pi}_{A_i}) = V(\bar{\boldsymbol{\pi}}_{A_i} \cdot U^{A_i}, \boldsymbol{\pi}_{A_i}) \leq V(\bar{\boldsymbol{\pi}}_{A_i} \cdot U^{A_{i+1}}, \boldsymbol{\pi}_{A_i})$$
$$\leq V(\boldsymbol{\pi}_{A_i} \cdot U^{A_{i+1}}, \boldsymbol{\pi}_{A_i}) \leq V(\boldsymbol{\pi}_{A_{i+1}} \cdot U^{A_{i+1}}, \boldsymbol{\pi}_{A_{i+1}}) = V(\bar{\boldsymbol{\pi}}_{A_{i+1}} \cdot U^{A_{i+1}}, \boldsymbol{\pi}_{A_{i+1}})$$

1st $\leq$ is strict if $\bar{\boldsymbol{\pi}}_{A_i} \cdot U^{A_i} < \bar{\boldsymbol{\pi}}_{A_i} \cdot U^{A_{i+1}}$ (by strict monot in 1st arg of V)

If $\bar{\boldsymbol{\pi}}_{A_i} \cdot U^{A_i} < \bar{\boldsymbol{\pi}}_{A_i} \cdot U^{A_{i+1}}$, we obtain a cycle and $\bar{\boldsymbol{\pi}}_{A_i} \cdot U^{A_i} < \bar{\boldsymbol{\pi}}_{A_i} \cdot U^{A_i}$, contradiction

- CIA rep w/ properties $\impliedby$ GACI + NIAS: application of Afriat's thm (see paper)

Unobservables

What if u/μ are unknown? Know that $u \in \mathcal{U}/\mu \in \mathcal{M}$: test NIAS and GACI for each u/μ

Easy-to-check conditions (linear conditions)

Overview

1. Costly Information Acquisition

2. Non-Separable Costs

3. Additive Costs
   - No Improving Attention Cycles (NIAC)
   - Optimal Information Structure with Additive Costs
   - Distinguishing Nonseparable and Additive Costs

4. Multiplicative Costs

5. Constrained Information Acquisition

6. Relation to Stochastic Choice

7. Entropy Costs

8. Limitations of and Alternatives to Entropy Costs

9. Implications of CIA for Choice

Additive Cost of Information

Cost of information structure $K : \Pi \to \mathbb{R} \cup \{\infty\}$ with $K(\pi) < \infty$ for some $\pi \in \Pi$

$\mathcal{K}$: set of all such cost functions

Additive cost: with loss.

Infinite costs = hard constraints.

e.g., bound on info acquired, restrictions on class of information structures.

Optimal info structures

$G(A, \pi) - K(\pi) = \pi \cdot U^A - K(\pi)$

$$\Pi^*(K, A) := \arg\max_{\pi \in \Pi} \pi \cdot U^A - K(\pi)$$

Optimality of Information Structures

Idea:

$$\begin{cases} \boldsymbol{\pi}_A \cdot U^A - K(\boldsymbol{\pi}_A) \geq \boldsymbol{\pi}_B \cdot U^A - K(\boldsymbol{\pi}_B) \\ \boldsymbol{\pi}_B \cdot U^B - K(\boldsymbol{\pi}_B) \geq \boldsymbol{\pi}_A \cdot U^B - K(\boldsymbol{\pi}_A) \end{cases} \Longrightarrow \boldsymbol{\pi}_A \cdot U^A + \boldsymbol{\pi}_B \cdot U^B \geq \boldsymbol{\pi}_B \cdot U^A + \boldsymbol{\pi}_A \cdot U^B$$

$$\begin{aligned} \Longleftrightarrow \quad \bar{\boldsymbol{\pi}}_A \cdot U^A + \bar{\boldsymbol{\pi}}_B \cdot U^B &= \boldsymbol{\pi}_A \cdot U^A + \boldsymbol{\pi}_B \cdot U^B \\ &\geq \boldsymbol{\pi}_B \cdot U^A + \boldsymbol{\pi}_A \cdot U^B \\ &\geq \bar{\boldsymbol{\pi}}_B \cdot U^A + \bar{\boldsymbol{\pi}}_A \cdot U^B \end{aligned}$$

where last $\geq$ due to $\boldsymbol{\pi}_{A_i} \geq_B \bar{\boldsymbol{\pi}}_{A_i}$

Interpretation: assignment problem

have $\{(\boldsymbol{\pi}_A, U^A)\}_{A \in \mathcal{D}}$; $\boldsymbol{\pi}_i \cdot U^j$: surplus generated by assigning i to j; $K(\boldsymbol{\pi}_i)$: cost

Surplus maximised by assigning i to i

Optimality of Information Structures

Definition

Given $\mu, u, (\mathcal{D}, P)$ sat. **no improving attention cycles** (NIAC) if for all sequences $(\bar{\pi}_{A_1}, U^{A_1}), (\bar{\pi}_{A_2}, U^{A_2}), ..., (\bar{\pi}_{A_k}, U^{A_k})$ with $A_i \in \mathcal{D}$, $\sum_{i=1}^{k} \bar{\pi}_{A_i} \cdot U^{A_i} \geq \sum_{i=1}^{k} \bar{\pi}_{A_i} \cdot U^{A_{i+1}} \pmod{k}$

Easy to check that $\neg$ GACI $\implies$ $\neg$ NIAC

Representation Result

Theorem (Caplin & Dean, '15 AER)

Fix μ and u and let $(\mathcal{D}, P)$ be a state-dependent stochastic choice dataset. The following are equivalent:

(i) $(\mathcal{D}, P)$ sat. NIAS and NIAC.

(ii) $(\mathcal{D}, P)$ admits an additive costly information acquisition representation.

(iii) $(\mathcal{D}, P)$ admits an additive costly information acquisition representation with $K \in \mathcal{K}$ nondecreasing in Blackwell order, convex.

Restrictions on K come for free; i.e., are untestable.

Can also normalise K s.t. zero cost for uninformative info structures.

Representation Result

Proof Sketch

- Additive CIA rep w/ properties $\implies$ NIAS, NIAC easier

focus instead on:

- NIAC + NIAS $\implies$ Additive CIA rep w/ properties.

$$\boldsymbol{\pi} \cdot U^A - K(\boldsymbol{\pi}) \leq \bar{\boldsymbol{\pi}}_A \cdot U^A - K(\bar{\boldsymbol{\pi}}_A) \iff K(\bar{\boldsymbol{\pi}}_A) - K(\boldsymbol{\pi}) \leq \bar{\boldsymbol{\pi}}_A \cdot U^A - \boldsymbol{\pi} \cdot U^A$$

Implies upper & lower bounds on cost difference of revealed info structures

$$\bar{\boldsymbol{\pi}}_A \cdot U^B - \bar{\boldsymbol{\pi}}_B \cdot U^B \leq K(\bar{\boldsymbol{\pi}}_A) - K(\bar{\boldsymbol{\pi}}_B) \leq \bar{\boldsymbol{\pi}}_A \cdot U^A - \bar{\boldsymbol{\pi}}_B \cdot U^A$$

Representation Result

Proof Sketch

- NIAC + NIAS $\implies$ Additive CIA rep w/ properties

  Implementation: WT find transfer $K : \cup_{A \in \mathcal{D}} \{\bar{\pi}_A\} \to \mathbb{R} \cup \{\infty\}$ s.t. makes info choice IC:

$$\bar{\pi}_A \cdot U^A - K(\bar{\pi}_A) \geq \bar{\pi}_B \cdot U^A - K(\bar{\pi}_B)$$

  $\forall A, B \in \mathcal{D}$; NIAC provides condition (Rockafellar '70)

  Pick arbitrary A_0; let

  $T(A) := \sup_{\text{all chains } A_0 \text{ to } A_k = A} \sum_{n=0}^{k-1} \bar{\pi}_{A_i} \cdot U^{A_{i+1}} - \bar{\pi}_{A_i} \cdot U^{A_i}$

▷ Skip

Representation Result

Proof Sketch

- NIAC + NIAS $\implies$ Additive CIA rep w/ properties

Pick arbitrary A_0; let

$T(A) := \sup_{\text{all chains } A_0 \text{ to } A_k = A} \sum_{n=0}^{k-1} \bar{\pi}_{A_n} \cdot U^{A_{n+1}} - \bar{\pi}_{A_n} \cdot U^{A_n}$

NIAC implies (i) $T(A_0) = 0$; **(ii)** $T(A_0) \geq T(A_k) + \bar{\pi}_{A_k} \cdot U^{A_0} - \bar{\pi}_{A_k} \cdot U^{A_k}$

$$\begin{aligned} T(A_k) + \bar{\pi}_{A_k} \cdot U^{A_0} - \bar{\pi}_{A_k} \cdot U^{A_k} &= (\bar{\pi}_{A_0} \cdot U^{A_1} - \bar{\pi}_{A_0} \cdot U^{A_0}) + (\bar{\pi}_{A_1} \cdot U^{A_2} - \bar{\pi}_{A_1} \cdot U^{A_1}) + \cdots \\ &\quad + (\bar{\pi}_{A_{k-1}} \cdot U^{A_k} - \bar{\pi}_{A_{k-1}} \cdot U^{A_{k-1}}) + (\bar{\pi}_{A_k} \cdot U^{A_0} - \bar{\pi}_{A_k} \cdot U^{A_k}) \\ &= -\sum_{n=0}^{k} \left(\bar{\pi}_{A_n} \cdot U^{A_n} - \bar{\pi}_{A_n} \cdot U^{A_{n+1 \,(\text{mod } k+1)}} \right) \leq 0 = T(A_0) \end{aligned}$$

i.e. $T(A_i)$ is bounded from above.

▷ Skip

Representation Result

Proof Sketch

- NIAC + NIAS $\implies$ Additive CIA rep w/ properties

Pick arbitrary A_0; let

$T(A) := \sup_{\text{all chains } A_0 \text{ to } A_k = A} \sum_{n=0}^{k-1} \bar{\pi}_{A_n} \cdot U^{A_{n+1}} - \bar{\pi}_{A_n} \cdot U^{A_n}$

NIAC implies (iii) $\forall A_i, A_j$, $T(A_i) \geq T(A_j) + \bar{\pi}_{A_j} \cdot U^{A_i} - \bar{\pi}_{A_j} \cdot U^{A_j}$

Suppose for chain $(A_0, A_1, ..., A_k)$ with $A_k = A_j$, $T(A_j) := \sum_{n=0}^{k-1} \bar{\pi}_{A_n} \cdot U^{A_{n+1}} - \bar{\pi}_{A_n} \cdot U^{A_n}$

If $\nexists \ell : A_\ell = A_i$, then set $A_{k+1} = A_i$ and note

$T(A_i) \geq \sum_{n=0}^{k-1} \left(\bar{\pi}_{A_n} \cdot U^{A_{n+1}} - \bar{\pi}_{A_n} \cdot U^{A_n} \right) + \bar{\pi}_{A_j} \cdot U^{A_i} - \bar{\pi}_{A_j} \cdot U^{A_j} = \sum_{n=0}^{k} \left(\bar{\pi}_{A_n} \cdot U^{A_{n+1}} - \bar{\pi}_{A_n} \cdot U^{A_n} \right)$

▷ Skip

Proof Sketch

- NIAC + NIAS $\implies$ Additive CIA rep w/ properties

Pick arbitrary A_0; let

$T(A) := \sup_{\text{all chains } A_0 \text{ to } A_k = A} \sum_{n=0}^{k-1} \bar{\pi}_{A_n} \cdot U^{A_{n+1}} - \bar{\pi}_{A_n} \cdot U^{A_n}$

NIAC implies (iii) $\forall A_i, A_j$, $T(A_i) \geq T(A_j) + \bar{\pi}_{A_j} \cdot U^{A_i} - \bar{\pi}_{A_j} \cdot U^{A_j}$

Suppose for chain $(A_0, A_1, ..., A_k)$ with $A_k = A_j$, $T(A_j) := \sum_{n=0}^{k-1} \bar{\pi}_{A_n} \cdot U^{A_{n+1}} - \bar{\pi}_{A_n} \cdot U^{A_n}$

If $\exists \ell : A_\ell = A_i$, then $T(A_i) \geq \sum_{n=0}^{\ell-1} \left(\bar{\pi}_{A_n} \cdot U^{A_{n+1}} - \bar{\pi}_{A_n} \cdot U^{A_n} \right)$

Then

$$T(A_j) - T(A_i) \leq \sum_{n=\ell}^{k-1} \left(\bar{\pi}_{A_n} \cdot U^{A_{n+1}} - \bar{\pi}_{A_n} \cdot U^{A_n} \right)$$

$$\iff T(A_j) - T(A_i) + (\bar{\pi}_{A_j} \cdot U^{A_i} - \bar{\pi}_{A_j} \cdot U^{A_j})$$

$$\leq \sum_{n=\ell}^{k-1} \left(\bar{\pi}_{A_n} \cdot U^{A_{n+1}} - \bar{\pi}_{A_n} \cdot U^{A_n} \right) + (\bar{\pi}_{A_j} \cdot U^{A_i} - \bar{\pi}_{A_j} \cdot U^{A_j})$$

$$= (\bar{\pi}_{A_i} \cdot U^{A_{\ell+1}} - \bar{\pi}_{A_i} \cdot U^{A_i}) + \cdots + (\bar{\pi}_{A_{k-1}} \cdot U^{A_j} - \bar{\pi}_{A_{k-1}} \cdot U^{A_{k-1}}) + (\bar{\pi}_{A_j} \cdot U^{A_i} - \bar{\pi}_{A_j} \cdot U^{A_j})$$

$$\leq 0 \qquad \text{(by NIAC)}$$

Representation Result

Proof Sketch

- NIAC + NIAS $\implies$ Additive CIA rep w/ properties

  Implementation: WT find transfer $K : \cup_{A \in \mathcal{D}} \{\bar{\pi}_A\} \to \mathbb{R} \cup \{\infty\}$ s.t. makes info choice IC:

$$\bar{\pi}_A \cdot U^A - K(\bar{\pi}_A) \geq \bar{\pi}_B \cdot U^A - K(\bar{\pi}_B)$$

  $\forall A, B \in \mathcal{D}$; NIAC provides condition (Rockafellar '70).

  Pick arbitrary A_0; let

  $T(A) := \sup_{\text{all chains } A_0 \text{ to } A_k = A} \sum_{n=0}^{k-1} \bar{\pi}_{A_n} \cdot U^{A_{n+1}} - \bar{\pi}_{A_n} \cdot U^{A_n}$

  NIAC implies $\forall A_i, A_j, \quad T(A_i) \geq T(A_j) + \bar{\pi}_{A_j} \cdot U^{A_i} - \bar{\pi}_{A_j} \cdot U^{A_j}$

  Set $K(\bar{\pi}_{A_j}) := \bar{\pi}_{A_j} \cdot U^{A_j} - T(A_j)$.

▷ Skip

Representation Result

Proof Sketch

- NIAC + NIAS $\implies$ Additive CIA rep w/ properties

> Extend K to mixtures of revealed info structures.

Define $S := \cup_{A \in \mathcal{D}} S(\bar{\pi}_A)$ and consider $\bar{\pi}_A : \Theta \to \Delta(S)$.

Define $co(\{\bar{\pi}_A \mid A \in \mathcal{D}\}) := \{\pi \in \Pi \mid \pi = \sum_{A \in \mathcal{D}} \lambda(A)\bar{\pi}_A \text{ for some } \lambda \in \Delta(\mathcal{D})\}$.

Extend K to $co(\{\bar{\pi}_A \mid A \in \mathcal{D}\})$ as $K(\sum_A \lambda(A)\bar{\pi}_A) = \sum_A \lambda(A)K(\bar{\pi}_A)$.

Representation Result

Proof Sketch

- NIAC + NIAS $\implies$ Additive CIA rep w/ properties

> Extend K to garblings of mixtures.

Let $\bar{\Pi}$ be the set of garblings of mixtures. For any $\pi' \in \mathcal{B}co(\{\bar{\pi}_A \mid A \in \mathcal{D}\})$, define

$$K(\pi') := \min\{K(\pi) \mid \pi \in co(\{\bar{\pi}_A \mid A \in \mathcal{D}\}) \text{ and } \pi \text{ is sufficient for } \pi'\}.$$

> Extend K to Π: set $K(\pi) := \infty$ for all $\pi \in \Pi \setminus \bar{\Pi}$.

Easy to check that IC still satisfied: $\pi' \in \bar{\Pi}$ less informative than some $\pi = \sum_A \lambda(A)\bar{\pi}_A$ and with $K(\pi') = K(\pi)$

$\implies \pi' \cdot U^B - K(\pi') = \pi' \cdot U^B - K(\pi) \leq \pi \cdot U^B - K(\pi) \leq \bar{\pi}_B U^B - K(\bar{\pi}_B)$ for any $B \in \mathcal{D}$.

> K convex and monotone in Blackwell order;
normalisation $K(\pi) = 0$ for uninformative structures is straightforward.

Representation Result

Proof Sketch

- NIAC + NIAS $\implies$ Additive CIA rep w/ properties

> Last step: construct $\boldsymbol{\sigma}_A : S(\boldsymbol{\pi}_A) \to \Delta(A)$ s.t. optimal choices

$\forall m \in S(\boldsymbol{\pi}_A)$, let

$$\boldsymbol{\sigma}_A(a|m) := \begin{cases} \frac{P_A(a)}{\sum_{b \in A \mid \bar{m}^b = m} P_A(b)}, & \text{if } m = \bar{m}^a \\ 0, & \text{if otherwise} \end{cases}$$

Note NIAS ensures $\boldsymbol{\sigma}_A$ is optimal

$\sum_{\theta} \boldsymbol{\mu}(\theta) P_A(a \mid \theta)[u(a,\theta) - u(b,\theta)] \geq 0 \implies \sum_{\theta} \bar{m}^a(\theta)[u(a,\theta) - u(b,\theta)] \geq 0.$

> Can show that $\boldsymbol{\sigma}_A \circ \bar{\boldsymbol{\pi}}_A = P_A$ by construction: data are matched.

Done! □

Optimal Information Structure with Additive Costs

DM chooses (π, σ)

(1) Distrib actions given beliefs: $\sigma : \Delta(\Theta) \to \Delta(A)$

$$\Sigma^*(A) := \left\{ \text{supp}(\sigma(m)) \in \arg\max_{a \in A} \mathbb{E}_m[u(a,\theta)],\ \forall m \in \Delta(\Theta) \right\}$$

(2) Distrib posterior beliefs given state: $\pi : \Theta \to \Delta(\Delta(\Theta))$ s.t. posterior distrib. MPS of prior: $\sum_\theta \mu(\theta) \sum_m \pi(m|\theta) m = \mathbb{E}_\mu[\mathbb{E}_{\pi(\cdot|\theta)}[m]] = \mu$.

$$\Pi^*(A, \sigma) := \arg\max_{\pi \in \Pi} \mathbb{E}_\mu[\mathbb{E}_{\pi(\cdot|\theta)}[\mathbb{E}_m[u(\sigma(m), \theta)]]] - K(\pi; \mu)$$

Lemma (Matejka & McKay '15 AER)

If K is strictly (weakly) increasing in $\geq_B$ (it is wloo that) $\sigma \in \Sigma^*(A), \pi \in \Pi^*(A, \sigma)$ sat.: $\nexists a, m, m'$ s.t. ((i) $\sigma(a|m) \in (0,1)$, and (ii)) $\sigma(a|m)\pi(m), \sigma(a|m')\pi(m') > 0$, and $m \neq m'$

Intuition: at most one posterior belief per action chosen with positive probability; deterministic tie-breaking.

Optimal Information Structure with Additive Costs

Proof

Suppose not. Then, $\exists a, m, m'$ s.t. $\sigma(a|m)\pi(m), \sigma(a|m')\pi(m') > 0$, and $m \neq m'$.

(1) Defining garbled signal: $m'' := \delta m + (1-\delta)m'$

Note: $\tilde{m}(\theta) = \dfrac{\mu(\theta)\pi(\tilde{m}|\theta)}{\pi(\tilde{m})}$. Let $\delta := \dfrac{\pi(m)}{\pi(m)+\pi(m')}$

Define $\pi' : \Theta \to \Delta(\Delta(\Theta))$:

$\pi'(\tilde{m}|\theta) := \pi(\tilde{m}|\theta) \quad \forall \tilde{m} \neq m, m'$, and $\pi'(m''|\theta) = \pi'(m|\theta) + \pi'(m'|\theta)$

Then $m''(\theta) = \dfrac{\mu(\theta)(\pi(m|\theta)+\pi(m'|\theta))}{\pi(m)+\pi(m')} = \dfrac{\mu(\theta)\pi'(m''|\theta)}{\pi'(m'')} = \delta m(\theta) + (1-\delta)m'(\theta)$.

(2) Defining strategy: $\sigma' : \Delta(\Theta) \to \Delta(A)$ s.t. $\sigma'(\tilde{m}) = \sigma(\tilde{m}) \; \forall \tilde{m} \neq m''$, and $\sigma'(a|m'') = 1$

(3) Verifying $\sigma' \in \Sigma^*(A)$

For $\tilde{m} = m, m'$, $\sigma(a|\tilde{m}) > 0$, $\mathbb{E}_{\tilde{m}}[u(a,\theta)] \geq \mathbb{E}_{\tilde{m}}[u(a',\theta)] \; \forall a' \in A$

By linearity, same holds for $m'' = \delta m + (1-\delta)m'$ and $\sigma' \in \Sigma^*(A)$

Optimal Information Structure with Additive Costs

Proof

Suppose not. Then, $\exists a, m, m'$ s.t. $\sigma(a|m)\pi(m), \sigma(a|m')\pi(m') > 0$, and $m \neq m'$

(1) Defining garbled signal: $m'' := \delta m + (1-\delta)m'$

(2) Defining strategy: $\sigma' : \Delta(\Theta) \to \Delta(A)$ s.t. $\sigma'(\tilde{m}) = \sigma(\tilde{m})\ \forall \tilde{m} \neq m''$, and $\sigma'(a|m'') = 1$

(3) Verifying $\sigma' \in \Sigma^*(A)$; (4) Verifying $\pi' \in \Pi^*(A, \sigma')$

$$\pi(m)\mathbb{E}_m[u(\sigma(m),\theta)] + \pi(m')\mathbb{E}_{m'}[u(\sigma(m'),\theta)] = \pi'(m'')\mathbb{E}_{m''}[u(\sigma'(m''),\theta)]$$

$$G^A(\pi) = \sum_{\tilde{m}\neq m,m'} \pi(\tilde{m})\mathbb{E}_{\tilde{m}}[u(\sigma(\tilde{m}),\theta)] + \sum_{\tilde{m}=m,m'} \pi(\tilde{m})\mathbb{E}_{\tilde{m}}[u(\sigma(\tilde{m}),\theta)]$$

$$= \sum_{\tilde{m}\neq m''} \pi'(\tilde{m})\mathbb{E}_{\tilde{m}}[u(\sigma'(\tilde{m}),\theta)] + \pi'(m'')\mathbb{E}_{m''}[u(\sigma'(m''),\theta)] = G^A(\pi')$$

Optimal Information Structure with Additive Costs

Proof

Suppose not. Then, $\exists a, m, m'$ s.t. $\boldsymbol{\sigma}(a|m)\boldsymbol{\pi}(m), \boldsymbol{\sigma}(a|m')\boldsymbol{\pi}(m') > \mathbf{0}$, and $m \neq m'$

(1) Defining garbled signal: $m'' := \delta m + (\mathbf{1} - \delta)m'$

(2) Defining strategy: $\boldsymbol{\sigma}' : \Delta(\Theta) \to \Delta(A)$ s.t. $\boldsymbol{\sigma}'(\tilde{m}) = \boldsymbol{\sigma}(\tilde{m})\ \forall \tilde{m} \neq m''$, and $\boldsymbol{\sigma}'(a|m'') = \mathbf{1}$

(3) Verifying $\boldsymbol{\sigma}' \in \boldsymbol{\Sigma}^*(A)$; (4) Verifying $\boldsymbol{\pi}' \in \boldsymbol{\Pi}^*(A, \boldsymbol{\sigma}')$

$G^A(\boldsymbol{\pi}) = G^A(\boldsymbol{\pi}')$

As $\boldsymbol{\pi} >_B \boldsymbol{\pi}'$ and K is increasing in $\geq_B$, $G^A(\boldsymbol{\pi}) - K(\boldsymbol{\pi}) \leq G^A(\boldsymbol{\pi}') - K(\boldsymbol{\pi}')$

If K strictly increasing, contradicts optimality of $\boldsymbol{\pi}$

Result extends to $\Theta \subseteq \mathbb{R}^n$

Distinguishing Multiplicative and Additive Costs

Menu (= decision problem) A yields **higher gross return from information** than menu B if $\boldsymbol{\pi}, \boldsymbol{\pi}'$ s.t. $\boldsymbol{\pi} >_B \boldsymbol{\pi}'$, $\boldsymbol{\pi} \cdot U^A - \boldsymbol{\pi}' \cdot U^A > \boldsymbol{\pi} \cdot U^B - \boldsymbol{\pi}' \cdot U^B$
($\iff$ $U^A - U^B : \Delta(\Theta) \to \mathbb{R}$ is strictly convex)

Proposition (Chambers, Liu, & Rehbeck, '20 JET)

Fix $\boldsymbol{\mu}$ and $u \geq \mathbf{0}$ and let $(\mathcal{D}, P)$ be a state-dependent stochastic choice dataset with $\mathcal{D} = \{A, B\}$. Suppose A yields higher gross return from information than B.

(i) If $\bar{\boldsymbol{\pi}}_B >_B \bar{\boldsymbol{\pi}}_A$, then $(\mathcal{D}, P)$ cannot sat. NIAC.

(ii) If $U^A > U^B$, $\bar{\boldsymbol{\pi}}_B >_B \bar{\boldsymbol{\pi}}_A$, and $(\mathcal{D}, P)$ sat. NIAS, then it sat. GACI

This is property that fails in their experiment. Still unclear if one should abandon additive separability because of that.

Intuition (my reading):

(i) Additive costs: if higher return to info at A than B, then cannot choose + informative info structure at B than A.

(ii) General nonseparable costs: people can do weird things.

Overview

1. Costly Information Acquisition

2. Non-Separable Costs

3. Additive Costs

4. Multiplicative Costs
   - Homothetic Axiom of Costly Information (HACI)
   - Representation Result

5. Constrained Information Acquisition

6. Relation to Stochastic Choice

7. Entropy Costs

8. Limitations of and Alternatives to Entropy Costs

9. Implications of CIA for Choice

Multiplicative Cost of Information

Cost of information structure $R : \Pi \to \mathbb{R}_+$ with $R(\pi) > 0$ for some $\pi \in \Pi$

$\mathcal{R}$: set of all such cost functions.

Multiplicative cost: with loss.

Lower R, higher cost; zero R as if infinite cost; model hard constraints.

e.g. bound on info acquired, restrictions on class of information structures.

Optimal info structures

$R(\pi)G(A, \pi) = R(\pi)\pi \cdot U^A$

$$\Pi^*(K, A) := \arg\max_{\pi \in \Pi} R(\pi)\pi \cdot U^A$$

Note: restrict $u \geq 0$ to make sense.

Optimality of Information Structures

Idea: Homothetic preferences!

Definition

Given μ and $u \geq 0$, let $\mathcal{D}_0 := \{A \in \mathcal{D} \mid \mathbb{E}_\mu[u(a,\theta) = 0, \forall a \in A]\}$. $(\mathcal{D}, P)$ sat. **homothetic axiom of costly information** (HACI) if for all sequences $(\bar{\pi}_{A_1}, U^{A_1}), (\bar{\pi}_{A_2}, U^{A_2}), ..., (\bar{\pi}_{A_k}, U^{A_k})$ with $A_i \in \mathcal{D} \setminus \mathcal{D}_0$, $\prod_{i=1}^{k} \frac{\bar{\pi}_{A_i} \cdot U^{A_{i+1}}}{\bar{\pi}_i \cdot U^{A_i}} \leq 1 \pmod{k}$

Representation Result

Theorem (Chambers, Liu, & Rehbeck, '20 JET)

Fix μ and $u \geq 0$ and let $(\mathcal{D}, P)$ be a state-dependent stochastic choice dataset. The following are equivalent:

(i) $(\mathcal{D}, P)$ sat. NIAS and HACI;

(ii) $(\mathcal{D}, P)$ admits a multiplicative costly information acquisition representation;

(iii) $(\mathcal{D}, P)$ admits a multiplicative costly information acquisition representation with $R \in \mathcal{R}$ nonincreasing in Blackwell order and concave;

Restrictions on R come for free; i.e. are untestable.

Can also normalise R s.t. $R(\pi) = 1$ for uninformative info structures.

Overview

1. Costly Information Acquisition

2. Non-Separable Costs

3. Additive Costs

4. Multiplicative Costs

5. Constrained Information Acquisition
   - Binary Axiom of Costly Information (BACI)
   - Representation Result

6. Relation to Stochastic Choice

7. Entropy Costs

8. Limitations of and Alternatives to Entropy Costs

9. Implications of CIA for Choice

Constrained Information Acqusition

Feasible information structures $\Pi_C \subseteq \Pi$ with $\Pi_C \neq \emptyset$.

Definition

Given μ and u, $(\mathcal{D}, P)$ sat. **binary axiom of costly information** (BACI) if for all $A, B \in \mathcal{D}$, $\bar{\pi}_A \cdot U^A \geq \bar{\pi}_B \cdot U^A$.

Note: BACI implies NIAC (same as $K(\pi) = 0$ if $\pi \in \Pi_C$ and $K(\pi) = \infty$ if ow) and HACI ($R(\pi) = 1_{\pi \in \Pi_C}$).

Representation Result

Theorem (Chambers, Liu, & Rehbeck, '20 JET)

Fix μ and u and let $(\mathcal{D}, P)$ be a state-dependent stochastic choice dataset. The following are equivalent:

(i) $(\mathcal{D}, P)$ sat. NIAS and BACI.

(ii) $(\mathcal{D}, P)$ admits a constrained information acquisition representation.

(iii) $(\mathcal{D}, P)$ admits a constrained information acquisition representation with $\mathbf{\Pi}_C$ convex.

wlog to consider $\mathbf{\Pi}_C$ convex

Overview

1. Costly Information Acquisition

2. Non-Separable Costs

3. Additive Costs

4. Multiplicative Costs

5. Constrained Information Acquisition

6. Relation to Stochastic Choice

7. Entropy Costs

8. Limitations of and Alternatives to Entropy Costs

9. Implications of CIA for Choice

10. Experimental Evidence

From Information Acquisition to Stochastic Choice

Stochastic choice: data looks random.

Random Utility

- Grand set of actions $\mathcal{A}$.
- Utility randomly drawn $u : \mathcal{A} \to \mathbb{R}$ from some set $\mathcal{U}$; DM chooses action to max realised utility.
  - Mistakes, trembles, etc.
- Can think $\Theta = \mathcal{U}$, $u(a, \theta) = u(a)$ for realised utility.
- Induces stochastic choice data $P_A : \Theta \to \Delta(A)$.
- *State-dependent* Menu-monotonicity: $P_A(a \mid \theta) \geq P_{A \cup \{b\}}(a \mid \theta)$
  - (usually lit refers to state-independent menu-motonicity).

CIA allows for violations of state-dependent menu-monotonicity, for all states.

Overview

1. Costly Information Acquisition

2. Non-Separable Costs

3. Additive Costs

4. Multiplicative Costs

5. Constrained Information Acquisition

6. Relation to Stochastic Choice

7. Entropy Costs
   - Introduction to Entropy
   - Characterising Choice with Entropy Costs
   - Entropy and Posterior Separable Costs
   - Entropy and MLRP-ordered Signals

8. Limitations of and Alternatives to Entropy Costs

Overview

1. Costly Information Acquisition

2. Non-Separable Costs

3. Additive Costs

4. Multiplicative Costs

5. Constrained Information Acquisition

6. Relation to Stochastic Choice

7. Entropy Costs
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Entropy

Particular functional form for information cost based on (Shanon) entropy.

Popular in applied literature and applications.

Originated in information theory (Shannon ‘48)
(good reference: Cover & Thomas, ‘06).

Entropy

Entropy: $H(X) := -\mathbb{E}_{P_X}[\ln P_X(X)]; \quad p \in \Delta(X)$
measure amount of information in X; convention $0 \ln 0 = 0$.

Joint Entropy: $H(X,Y) = -\mathbb{E}_{P_{X,Y}}[\ln P_{X,Y}(X,Y)]$.

$\mathbb{E}$ Conditional Entropy $H(Y|X) := \sum_x P_X(x) H(Y|x)$.
Note: if $X \perp\!\!\!\perp Y$, then $H(Y|X) = H(Y)$.

Entropy Chain Rule: $H(X,Y) = H(X) + H(Y|X)$; $H((X,Y)|Z) = H(X|Z) + H(Y|X,Z)$
$H(X_1,...,X_n) = H(X_1) + \sum_{i=2}^{n} H(X_i \mid X_1,...,X_{i-1})$.

Theorem (Khinchin, '57)

H is unique function defined on arbitrary finite-dimensional simplices that

(i) depends only on prob distr (not values): $H(X) = H(p)$;

(ii) max at uniform distr: $\max_{p \in \Delta^n} H(p) = H((1/n, 1/n, ..., 1/n))$;

(iii) is unaffected by adding zero prob state: $H((p_1, p_2, ..., p_n)) = H((p_1, p_2, ..., p_n, 0))$; and

(iv) is additive: $(X,Y) \sim P_{XY}$, $H(X,Y) = H(X) + H(Y|X)$.

KL and Mutual Information

Kullback–Leibler divergence (KL) $\equiv$ Relative Entropy: $D(p||q) = \mathbb{E}_p[\ln(p(X)/q(X))]$, with $\mathbf{0} \cdot \ln(\mathbf{0}/\mathbf{0}) = \mathbf{0} \ln \mathbf{0} = \mathbf{0}$, and $p \cdot \ln(p/\mathbf{0}) = \infty$ for $p > \mathbf{0}$

KL is basis of Bayesian inference!

$D(p||q) \geq \mathbf{0}$; w/ = iff $p = q$.

Not symmetric: $D(p||q)$ in general $\neq D(q||p)$.

$\mathbb{E}$ Conditional KL $D(p_{Y|X}||q_{Y|X}) := \mathbb{E}_{x\sim p_X}[D(p_{Y|X}||q_{Y|X})]$.

KL Chain Rule: $D(p_{X,Y}||q_{X,Y}) = D(p_X||q_X) + D(p_{Y|X}||q_{Y|X})$.

Entropy and Mutual Information

Mutual Information (MI): $I(X;Y) = D(p_{X,Y}||p_X p_Y) = \sum_{x,y} p_{X,Y}(x,y) \left[\ln\left(\frac{p_{X,Y}(x,y)}{p_X(x)p_Y(y)}\right)\right]$.

- how much info rv contains about another;
- reduction in uncertainty on rv due to knowledge of the other.

Relation to Entropy

$I(X;X) = H(X)$.

$I(X;Y) = I(Y;X) = H(X) - H(X|Y) = H(Y) - H(Y|X) = H(X) + H(Y) - H(X,Y)$.

Properties

Conditional MI: $I((X;Y)|Z) = H(X|Z) - H(X|Y,Z)$.

MI Chain Rule: $I(X_1,...,X_n;Y) = I(X_1;Y) + \sum_{i=2}^{n} I((X_i;Y)|X_1,...,X_{i-1})$.

$I(X;Y) \geq 0$; w/ = iff $X \perp\!\!\!\perp Y$.

Reduction in uncertainty about rv due to knowledge of the other.

Entropy

Convexity/Concavity

- Entropy $H(p)$ concave in p.
- KL $D(p||q)$ convex in (p, q).
- MI $I(X;Y)$ concave in p_X given $p_{Y|X}$:
  $I(X;Y) = H(Y) - H(Y|X) = H(\sum_x p_X(x)p_{Y|X}(\cdot|x)) - \sum_x p_X(x)H(p_{Y|X}(\cdot|x))$.
- MI $I(X;Y)$ convex in $p_{Y|X}$ given p_X:
  $I(X;Y) = D(p_{X,Y}||p_X p_Y) = D(p_{Y|X}p_X||\mathbb{E}_{p_X}[p_{Y|X}]p_X)$.

Entropy

More Properties

- Conditioning reduces entropy: $H(X) \geq H(X|Y) \iff I(X;Y) \geq \mathbf{0}$.
  But conditioning *can increase* MI: possible $I(X;Y) < I((X;Y)|Z)$, e.g. $X \perp\!\!\!\perp Y$ and $Z = X + Y$.
- Transformation reduces entropy and MI: $H(f(X)) \leq H(X); \quad I(X;Y) \geq I(X;f(Y))$.
- No uncertainty, zero entropy: $H(f(X)|X) = \mathbf{0}$ ($f(X)$ is known given X).
- Entropy of a sum: $X \perp\!\!\!\perp Y \implies H(X+Y) \geq \max\{H(X), H(Y)\}$ $H(X+Y|X) = H(Y|X)$.
- Independence bound on entropy: $H(X_1, ..., X_n) \leq \sum_{i=1}^{n} H(X_i)$ (via chain rule for H).

Entropy

Probability Bounds

if $X \perp\!\!\!\perp Y, X \sim p, Y \sim q$, then

$\mathbb{P}(X = Y) \geq \max\{\exp(-H(p) - D(p||q)), \exp(-H(q) - D(q||p))\}$

if X, Y iid, then $\mathbb{P}(X = Y) \geq \max\{\exp(-H(p)\}$; w/ = iff unif distr

Derived Measures

Metric: $\delta(X;Y) := H(X,Y) - I(X;Y)$

Correlation measure: $I(X;Y)/H(X)$

Statistics

$\mu \in \Delta(\Theta)$; $p_\theta \in \Delta(X)$; T statistic of data X

Note: $I(\theta; T(X)) \leq I(\theta; X)$

Sufficient statistic: T s.t. $I(\theta; T(X)) = I(\theta; X)$

Overview

1. Costly Information Acquisition

2. Non-Separable Costs

3. Additive Costs

4. Multiplicative Costs

5. Constrained Information Acquisition

6. Relation to Stochastic Choice

7. Entropy Costs
   - Introduction to Entropy
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8. Limitations of and Alternatives to Entropy Costs

Entropy Costs

Recall: CIA with additive costs

Given $\mu \in \Delta(\Theta)$ and $u : A \times \Theta \to \mathbb{R}$

$$\max_{\pi \in \Pi} \pi \cdot U^A - K(\pi)$$

Posterior beliefs: $m \sim \pi(\cdot \mid \theta)$

Entropy (MI) costs: $K(\pi; \mu) := \lambda I(\theta; m) = \lambda(H(\theta) - H(\theta|m))$

$\lambda > 0$: scalar that parameterises cost

Cost measures reduction in prior entropy due to information acquired

Fully uninformative π: $m = \mu$; $\theta|m = \theta$ and thus $H(\theta|m) = H(\theta)$, $I(\theta; m) = 0$

Fully informative π: m is suff statistic for θ; $H(\theta|m) = 0$, $I(\theta; m) = H(\theta)$

For any π: $H(\theta) \geq I(\theta; m) \geq 0$

Characterising Choice with Entropy Costs

Lemma (Matejka & McKay '15 AER)

If K is strictly (weakly) increasing in $\succeq_B$ (it is wloo that) $\boldsymbol{\sigma} \in \boldsymbol{\Sigma}^*(A), \boldsymbol{\pi} \in \boldsymbol{\Pi}^*(A, \boldsymbol{\sigma})$ sat.: $\nexists a, m, m'$ s.t. ((i) $\boldsymbol{\sigma}(a|m) \in (0, 1)$, and (ii)) $\boldsymbol{\sigma}(a|m)\boldsymbol{\pi}(m), \boldsymbol{\sigma}(a|m')\boldsymbol{\pi}(m') > 0$, and $m \neq m'$

Given $\boldsymbol{\pi}$, associate each $m \in S(\boldsymbol{\pi})$ with action a optimal at $m \in \Delta(\Theta)$; denoted m^a

$$\sum_{\theta}\sum_{m} U^A(m)\boldsymbol{\pi}(m|\theta)\boldsymbol{\mu}(\theta) = \sum_{\theta}\sum_{a} U^A(m^a)\boldsymbol{\pi}(m^a|\theta)\boldsymbol{\mu}(\theta)$$

$$= \sum_{a} U^A(m^a)\sum_{\theta}\boldsymbol{\pi}(m^a|\theta)\boldsymbol{\mu}(\theta) = \sum_{a}\sum_{\theta} u(a,\theta)m^a(\theta)P_A(a)$$

$$= \sum_{a}\sum_{\theta} u(a,\theta)P_A(\theta|a)P_A(a) = \sum_{\theta}\sum_{a} u(a,\theta)P_A(a|\theta)\boldsymbol{\mu}(\theta)$$

Can rewrite cost function:

$$K(\boldsymbol{\pi};\boldsymbol{\mu}) = \lambda I(\theta; m^a) = \lambda(H(\theta) - H(\theta|m^a)) = \lambda(H(m) - H(m|\theta))$$

$$= \lambda(H(P_A(a)) - H(P_A(a|\theta))) = \lambda\left(\sum_{\theta}\boldsymbol{\mu}(\theta)\sum_{a} P_A(a|\theta)\ln P_A(a|\theta) - \sum_{a} P_A(a)\ln P_A(a)\right)$$

Characterising Choice with Entropy Costs

$$\max_{\pi \in \Pi} \pi \cdot U^A - K(\pi; \mu) = \max_{P_A : \Theta \to \Delta(A)} \mathbb{E}_\mu[\mathbb{E}_{P_A(\cdot|\theta)}[u(a,\theta)]] - \lambda(H(P_A(a)) - \mathbb{E}_\mu[H(P_A(a|\theta))])$$

$$\text{s.t. } P_A(a) = \mathbb{E}_\mu[P_A(a|\theta)]$$

Theorem 1 (Matejka & McKay '15 AER)

P_A is optimal only if $P_A(a|\theta) = \dfrac{P_A(a)\exp(u(a,\theta)/\lambda)}{\sum_b P_A(b)\exp(u(b,\theta)/\lambda)}$ a.s.

Proposition 1 (Caplin, Dean & Leahy '19 REStud)

P_A is optimal iff sat. (1) MM condition, and (2) $\forall a \in A$

$$\sum_\theta \mu(\theta) \frac{\exp(u(a,\theta)/\lambda)}{\sum_b P_A(b)\exp(u(b,\theta)/\lambda)} \leq 1$$

with equality is $P_A(a) > 0$

Proof: constrained optimisation

Computing a Solution

Blahut-Arimoto Algorithm (Cover & Thomas '12)

Ensures convergence

$$P_A^{n+1}(a|\theta) = \frac{P_A^n(a)\exp(u(a,\theta)/\lambda)}{\sum_{b\in A} P_A^n(b)\exp(u(b,\theta)/\lambda)}$$

$$P_A^{n+1}(a) = \left(\sum_{\theta} \mu(\theta)\frac{\exp(u(a,\theta)/\lambda)}{\sum_{b\in A} P_A^n(b)\exp(u(b,\theta)/\lambda)}\right) P_A^n(a)$$

Uniqueness of a Solution with Entropy Costs

Vectors $\{x_i\}_i$, $x_i \in \mathbb{R}^n$ are affinely independent if

$\forall \boldsymbol{\lambda}_i \in \mathbb{R}$ s.t. $\sum_i \boldsymbol{\lambda}_i = \mathbf{0}$ and $\sum_i \boldsymbol{\lambda}_i x_i = \mathbf{0}$, then $\boldsymbol{\lambda}_i = \mathbf{0}$ $\forall i$

Implied by linear independence

Theorem 2 (Caplin & Dean '13 WP)

If $\{\{\exp(u(a, \boldsymbol{\theta})/\boldsymbol{\lambda})\}_{\boldsymbol{\theta}}, a \in A\}$ are affinely independent, there is a unique optimal $\boldsymbol{\pi}$.

Characterising Beliefs with Entropy Costs

Can also characterise optimal posterior beliefs

$\Longrightarrow$ Rewrite conditions in terms of m^a

Proposition 2 (Caplin, Dean, & Leahy '19 REStud)

π is optimal iff

(1) $\sum_a \pi(m^a)m^a = \mu$

(2) [Invariant LR for Chosen Actions] $\forall a, b : P_A(a), P_A(b) > 0$

$$\frac{m^a(\theta)}{\exp(u(a,\theta)/\lambda)} = \frac{m^b(\theta)}{\exp(u(b,\theta)/\lambda)}$$

(3) [LR for Unchosen Actions] $\forall a, b : P_A(a) > 0 = P_A(b)$

$$\sum_\theta \frac{m^a(\theta)}{\exp(u(a,\theta)/\lambda)} \exp(u(b,\theta)/\lambda) \leq 1$$

Endogenous Consideration Sets

Caplin, Dean, & Leahy ('19 REStud) study endogenous consideration sets $\{a \mid P_A(a) > 0\}$

- Needle in haystack (3.1): only 1 action good, all others bad
- Independent values (3.2): $u(a, \cdot) \perp\!\!\!\perp u(b, \cdot)$; finite utility levels

See also Jung, Kim, Matejka, & Sims ('19 REStud)

Overview

1. Costly Information Acquisition

2. Non-Separable Costs

3. Additive Costs

4. Multiplicative Costs

5. Constrained Information Acquisition

6. Relation to Stochastic Choice

7. Entropy Costs
   - Introduction to Entropy
   - Characterising Choice with Entropy Costs
   - Entropy and Posterior Separable Costs
   - Entropy and MLRP-ordered Signals

8. Limitations of and Alternatives to Entropy Costs

Posterior-Based Approach

Definition

$K(\pi;\mu)$ is a **posterior separable** cost function if for each $\mu \in \Delta(\Theta)$, $\exists k_\mu : \Delta(\Theta) \to \mathbb{R}\cup\{\infty\}$
s.t. $K(\pi;\mu) := \mathbb{E}_\pi[k_\mu(m)] - k_\mu(\mu)$
where k_μ is strictly convex and finite for $m \in \text{int}\Delta(\Theta)$, and $\pi(m) = \mathbb{E}_\mu[\pi(m|\theta)]$

$K(\pi;\mu)$ is a **uniformly posterior separable** cost function if $\exists k : \Delta(\Theta) \to \mathbb{R} \cup \{\infty\}$ s.t.
$\forall \mu \in \Delta(\Theta)$, $K(\pi;\mu) := \mathbb{E}_\pi[k(m)] - k(\mu)$
where k is strictly convex and finite for $m \in \text{int}\Delta(\Theta)$, and $\pi(m) = \mathbb{E}_\mu[\pi(m|\theta)]$

Caplin, Dean, & Leahy ('18 WP) provide representation results, building on Caplin & Dean ('15 AER)

Posterior-Based Approach

Definition

$K(\pi;\mu)$ **posterior separable**: $K(\pi;\mu) := \mathbb{E}_\pi[k_\mu(m)] - k_\mu(\mu)$

PS Costs = NIAS, NIAC (for additivity), technical restrictions (non-linearity, convexity, continuity, and a richness condition)

+ Separability Condition: makes cost of particular posterior independent from distrib of posteriors

Fix A and μ; P_A state-dep stochastic choice

$S(P_A)$:= actions taken with positive probability

(Revealed) posteriors for chosen actions m^a: $M(\mu, A)$

Can find problem $\tilde{A} = (A \setminus \{b\}) \cup \{c\}$ s.t. posteriors m^a the same for $a \neq b, c$

Posterior-Based Approach

Definition

$K(\pi;\mu)$ **uniformly posterior separable**: $K(\pi;\mu) := \mathbb{E}_\pi[k(m)] - k(\mu)$

UPS Costs = PS Costs + Locally Invariant Posteriors

Fix A and μ; $P_A : \Theta \to \Delta(A)$; $S(P_A)$:= actions taken with positive probability
(Revealed) posteriors for chosen actions m^a: $M(\mu, A)$

If revealed posteriors under μ still feasible under $\tilde{\mu}$ — i.e. $\tilde{\mu} \in \text{co}(M^A(\mu))$
then $M(\tilde{\mu}, A) = M(\mu, A)$

If different problem $\tilde{A} \subseteq S(P_A)$, but $\{m^a\}_{a\in\tilde{A}}$ still feasible given $\tilde{\mu}$,
then $M(\tilde{\mu}, A) = \{m^a\}_{a\in\tilde{A}}$

Only choice probabilities adjust so that $\tilde{\mu} = \sum_{a\in\tilde{A}} \tilde{P}_{\tilde{A}}(a) m^a$
Useful for problem-solving!

Posterior-Based Approach

Entropy Costs = UPS + Invariance Under Compression

States θ, θ' payoff equivalent if $u(a, \theta) = u(a, \theta')$ $\forall a$

Invariance under compression: DM doesn't waste resources in distinguishing between payoff equivalent states; i.e. treats θ and θ' the same

Reasonable? Appeals to maximiser in us, but

seems at odds with situation in which

(1) learning about state is informative about likelihood of nearby states,

(2) payoffs at states nearby θ may be very different from those near θ'

Overview

1. Costly Information Acquisition

2. Non-Separable Costs

3. Additive Costs

4. Multiplicative Costs

5. Constrained Information Acquisition

6. Relation to Stochastic Choice

7. Entropy Costs
   - Introduction to Entropy
   - Characterising Choice with Entropy Costs
   - Entropy and Posterior Separable Costs
   - Entropy and MLRP-ordered Signals

8. Limitations of and Alternatives to Entropy Costs

Entropy and MLRP-ordered Signals

A, Θ ordered sets; $|\Theta| \geq 3$

$u : A \times \Theta$ sat. **(strict) increasing differences** if $\forall a' > a, \theta' > \theta$,
$u(a', \theta') - u(a, \theta')(>) \geq u(a', \theta) - u(a, \theta)$

π sat. **MLRP** if $\forall m, m' \in S(\pi)$, either

$$\frac{m'(\theta')}{m'(\theta)} \geq \frac{m(\theta')}{m(\theta)}$$

whenever $\theta' > \theta$, or vice-versa

K sat. uniform posterior separability, smoothness, invariance to relabeling states

Theorem (Mensch '19 WP)

Take assumptions as given. π sat. MLRP for all u sat. incresing differences, and all priors $\mu \in \Delta(\Theta)$ if and only if information costs are entropic

Entropy and MLRP-ordered Signals

Theorem (Mensch '19 WP)

Take assumptions as given. π sat. MLRP for all u sat. incresing differences, and all priors $\mu \in \Delta(\Theta)$ if and only if information costs are entropic

Intuition for sufficiency:

At optimal, MM15 imply: $u(a', \theta) - u(a, \theta) = \lambda(\ln m^{a'}(\theta) - \ln m^{a}(\theta))$

$\forall \theta$ and a', a chosen with positive probability

Increasing differences: $(\ln m^{a'}(\theta') - \ln m^{a}(\theta')) - (\ln m^{a'}(\theta) - \ln m^{a}(\theta)) \geq 0$

$$\implies \frac{m^{a'}(\theta')}{m^{a'}(\theta)} \geq \frac{m^{a}(\theta')}{m^{a}(\theta)}$$

Entropy and MLRP-ordered Signals

Theorem (Mensch '19 WP)

Take assumptions as given. π sat. MLRP for all u sat. incresing differences, and all priors $\mu \in \Delta(\Theta)$ if and only if information costs are entropic

Provide foundation for typically assumed use of MLRP signals

Result extends to A, Θ compact subsets of $\mathbb{R}$, with μ absolutely continuous, u continuous

Applications to games (see sections 4, 5)

e.g. uniqueness of symmetric eqm of IPV 1PA and 2PA with 1st stage of info acq wrt own valuation, and eqm strategies sat. MLRP; revenue equiv holds

Overview

1. Costly Information Acquisition

2. Non-Separable Costs

3. Additive Costs

4. Multiplicative Costs

5. Constrained Information Acquisition

6. Relation to Stochastic Choice

7. Entropy Costs

8. Limitations of and Alternatives to Entropy Costs

9. Implications of CIA for Choice

10. Experimental Evidence

Limitations of Entropy Costs

Example

WT invest if return on risky asset r greater than return on risk-free asset r_0
with entropy, prob choosing best action depends only on whether $r \lessgtr r_0$, not on distance $r - r_0$

WT detect mistakes on data entry on volume of sales
with entropy, prob detecting mistake depends on whether or not there was a mistake, not on whether the mistake was GPB 1 or GPB 1 bn

No notion of similarity of states

"Entropy does not depend on a metric, i.e., the distance between states does not matter. With entropy, it is as difficult to distinguish the temperature of 10C from 20C, as 1C from 2C." (Mackowiak, Matejka, and Wiederholt, 2018)

Same holds for Tsallis entropy (Caplin, Dean, & Leahy, '18 WP; 8.3)

Limitations of Entropy Costs

Metric-dependent Additive Cost Functions

Pomatto, Strack, & Tamuz ('19 WP)

$$C(\pi) = \sum_{\theta,\tilde{\theta}\in\Theta} \beta_{\theta,\tilde{\theta}} D(\pi(\cdot \mid \theta)||\pi(\cdot \mid \tilde{\theta})), \quad \text{where } \beta_{\theta,\tilde{\theta}} \propto \frac{1}{(\theta-\tilde{\theta})^2}$$

Hebert & Woodford ('21 AER): Fisher information costs (limit result)

$$\mathbb{E}_{\mu}\left[\sum_{a|P_A(a)>0} \frac{\left(\frac{\partial}{\partial\theta} P_A(\theta|a)\right)^2}{P_A(\theta|a)}\right]$$

Other Formulations

Capacity constraints

Fixed information structures

As with utility functions (e.g. pervasiveness of CES and quasi-linear utility), whether assumptions on costs are reasonable or not, depends on application

Overview

1. Costly Information Acquisition

2. Non-Separable Costs

3. Additive Costs

4. Multiplicative Costs

5. Constrained Information Acquisition

6. Relation to Stochastic Choice

7. Entropy Costs

8. Limitations of and Alternatives to Entropy Costs

9. Implications of CIA for Choice

10. Experimental Evidence

Implications of CIA for Choice

Strictly dominated actions never chosen

(true in general for CIA).

Adding duplicates to action *a* doesn't affect prob. choosing other actions

(true in general for CIA when info is about θ; not necessarily true for search models).

Increasing payoffs to action *a* increases *unconditional* $P_A(a)$ under entropy costs

(sequential CIA with fixed info structures can drop unconditional: Gonçalves '23 WP).

MM15 fn 18: claim that with exogenous info structure, making action better can make it chosen less often in expectation (wrt μ)

However, in example info structure changes as well

If info structure is truly fixed, this never happens

Implications of CIA for Choice

CIA allows for violations of state-dependent menu-monotonicity, for all states

State-dependent menu-monotonicity: $P_A(a \mid \boldsymbol{\theta}) \geq P_{A\cup\{b\}}(a \mid \boldsymbol{\theta})$.

MM15, Prop. 6: Example of $P_A(a \mid \boldsymbol{\theta}) < P_{A\cup\{b\}}(a \mid \boldsymbol{\theta})$ $\forall \boldsymbol{\theta}$ with entropy costs.

Contrastingly: sequential CIA with fixed info structures, state-dependent menu-monotoniticity holds! (Gonçalves '23 WP).

Overview

1. Costly Information Acquisition

2. Non-Separable Costs

3. Additive Costs

4. Multiplicative Costs

5. Constrained Information Acquisition

6. Relation to Stochastic Choice

7. Entropy Costs

8. Limitations of and Alternatives to Entropy Costs

9. Implications of CIA for Choice

10. Experimental Evidence

Experimental Evidence

Choice in perception problems:

- Dewan & Neligh '20 JET: subjects mostly satisfy NIAS and NIAC, and mostly responsive to incentives in terms of performance.
- Dean & Neligh '20 WP: support for CIA, but violations of RUM.
- In general entropy poorly fitted for perceptual problems
- Caplin, Csaba, Leahy, & Nov '21 QJE: cool method to recover info costs experimentally.

Choosing information structures:

- Ambuehl & Li '15 GEB: undervalue info; beliefs underreact to info; biases due to nonstandard belief updating.
- Charness, Oprea, & Yuksel '21 JEEA: rely on heuristics esp. confirmation-seeking.
- Novak, Matveenko, & Ravaioli '21 WP: mostly in line with CIA, prob choosing best π depends on value difference, tend to prefer simpler π.

Overview

1. Costly Information Acquisition

2. Non-Separable Costs

3. Additive Costs

4. Multiplicative Costs

5. Constrained Information Acquisition

6. Relation to Stochastic Choice

7. Entropy Costs

8. Limitations of and Alternatives to Entropy Costs

9. Implications of CIA for Choice

10. Experimental Evidence

Applications

- Business cycle dynamics (Mackowiak & Wiederhold ‘15 REStud; Benhabib, Liu, & Wang ‘16 JET).
- Labor market discrimination (Bartos, Bauer, Chytilova, & Matejka ‘16 AER).
- Hiring (Acharya & Wee ‘20 AEJMacro).
- Choosing health insurance (Heiss, McFadden, & Winter ‘21 AER; Brown & Jeon ‘21 WP).
- Product pricing (Matejka ‘16 REStud; Martin ‘17 GEB; Stevens ‘20 REStud).
- Electoral competition (Matejka & Tabellini ‘21 JEEA; Feddersen & Sandroni ‘06 QJPS).
- Persuasion (Le Treust & Tomala ‘19 JET, Bloedel & Segal ‘18 WP; Wei ‘21 JMathE).
- Trade (Dasgupta & Mondria ‘18 JInternationalE).
- Principal-Agent problems (Lipnowski, Mathevet, & Wei ‘20 AERI).
- News media (Nimark & Pitschner ‘19 JET).
- Financial trading (Van Nieuwerburgh & Veldkamp ‘10 REStud; Kacperczyk, Van Nieuwerburgh & Veldkamp ‘16 Ecta; Vives & Yang ‘18 WP).

Applications

- Structural IO (Aguirregabiria & Jeon ‘20 RIO; Matveenko ‘17 WP; Fosgerau, Monardo, & De Palma ‘20 WP).
- Phillips curve (Afrouzi & Yang ‘21 WP).
- Bargaining (Ravid ‘20 AER).
- Monetary policy (Angeletos & La’O ‘20 JPE).
- Auctions (Persico ‘00 Ecta; Shi ‘12 GEB).